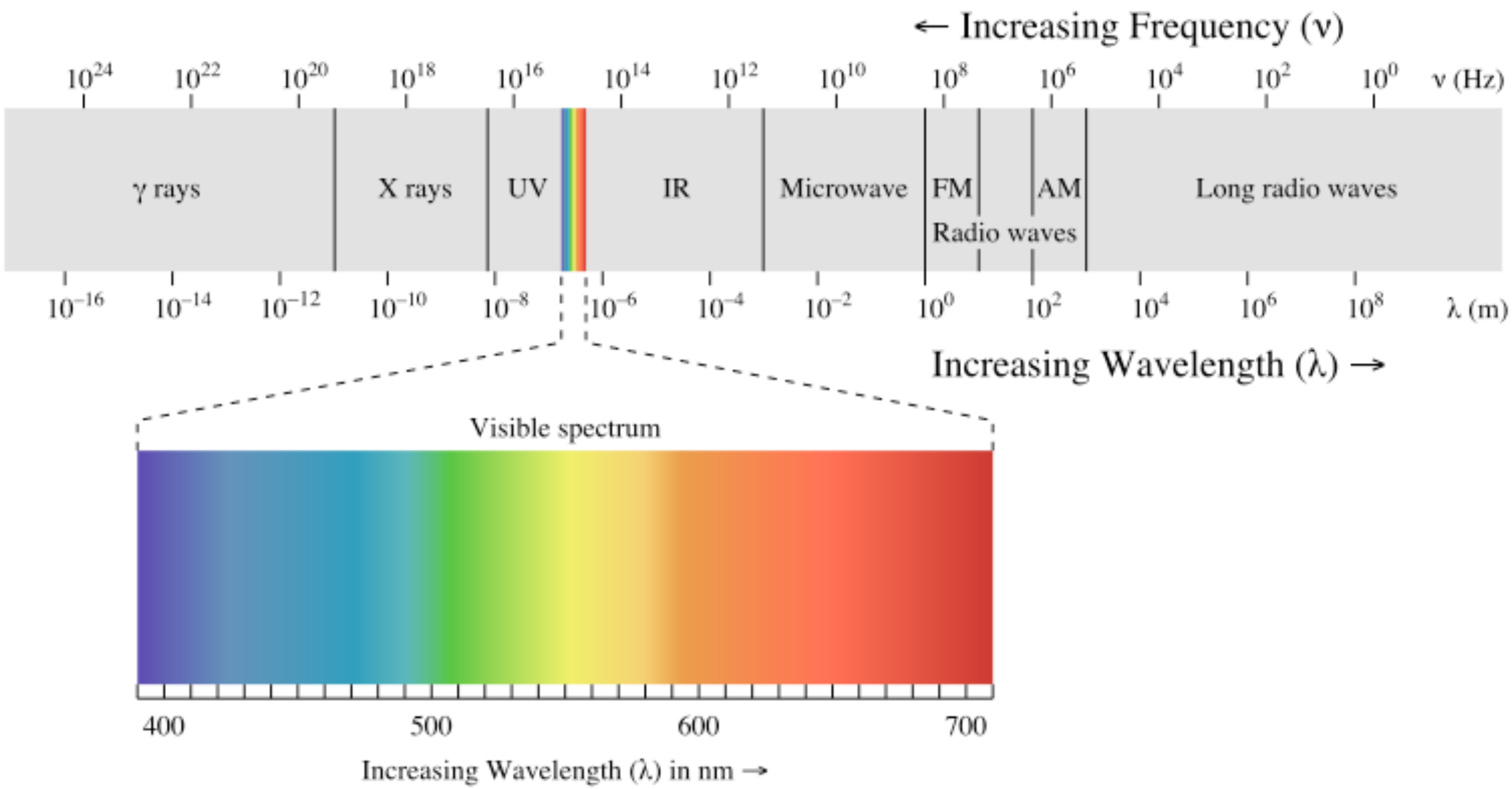
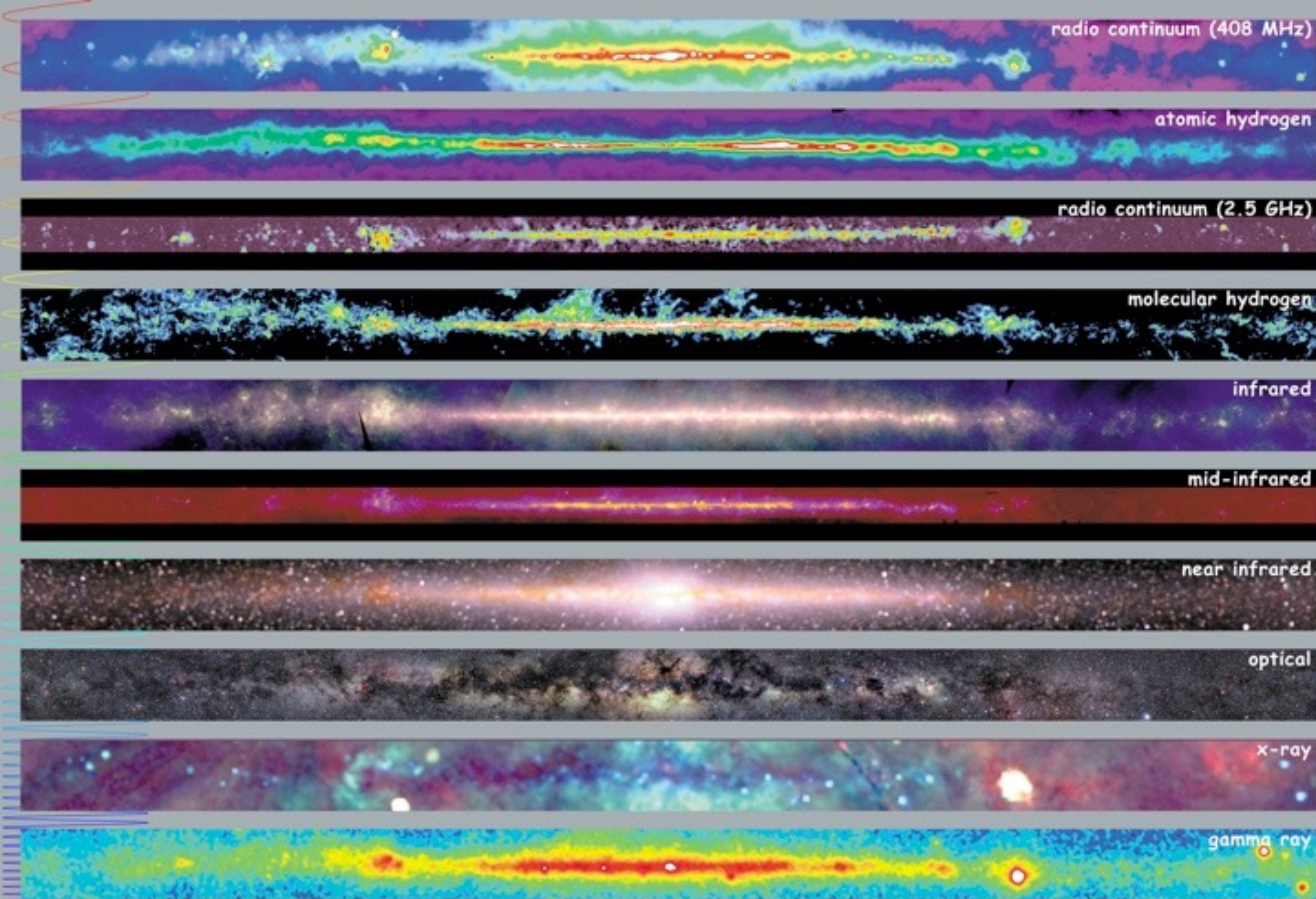
Introduction to *Radiation Transfer*

Hiroyuki Hirashita
(平下 博之)

1. Electromagnetic Radiation



All wavelengths are important in astronomy!

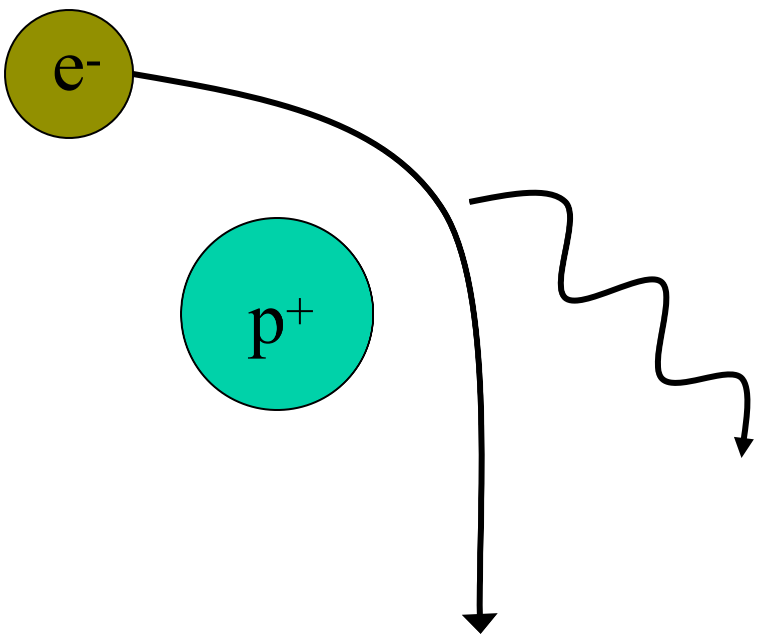


Multiwavelength Milky Way

Radiation Mechanisms

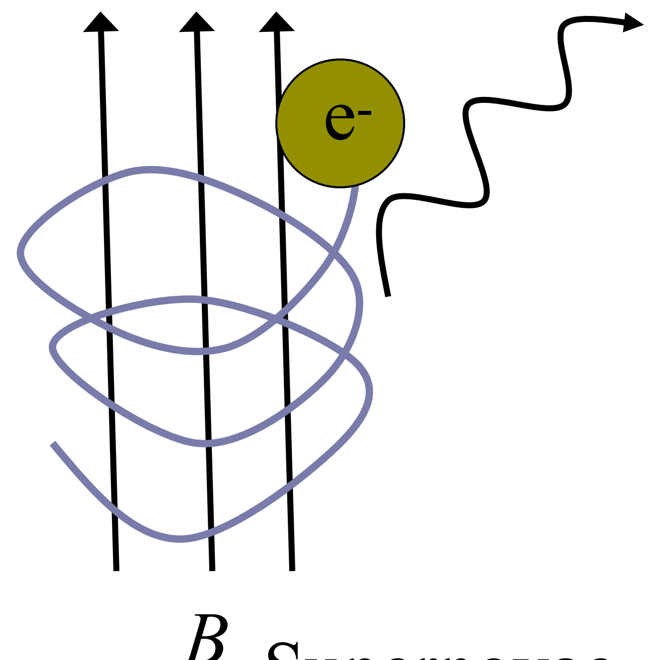
Radiation ⇔ Acceleration of electric charge

Bremsstrahlung (free–free)



Ionized gas:
e.g., H II region
Intracluster medium

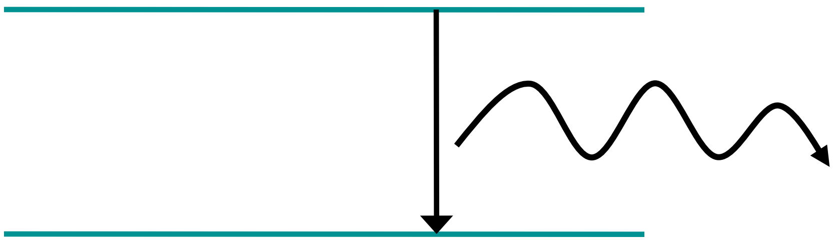
Synchrotron (magnetic bremsstrahlung)



Supernovae
AGN jet

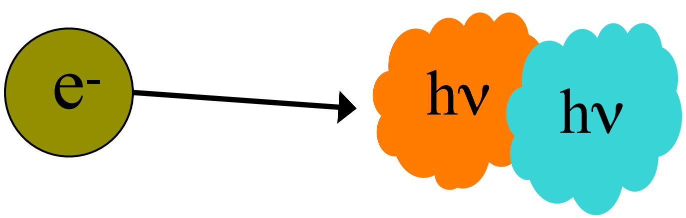
Radiation as Photons

Radiative transition



electric ~ UV, optical
molecular vibrational ~ near-IR
molecular rotational ~ radio

(Inverse) Compton Radiation

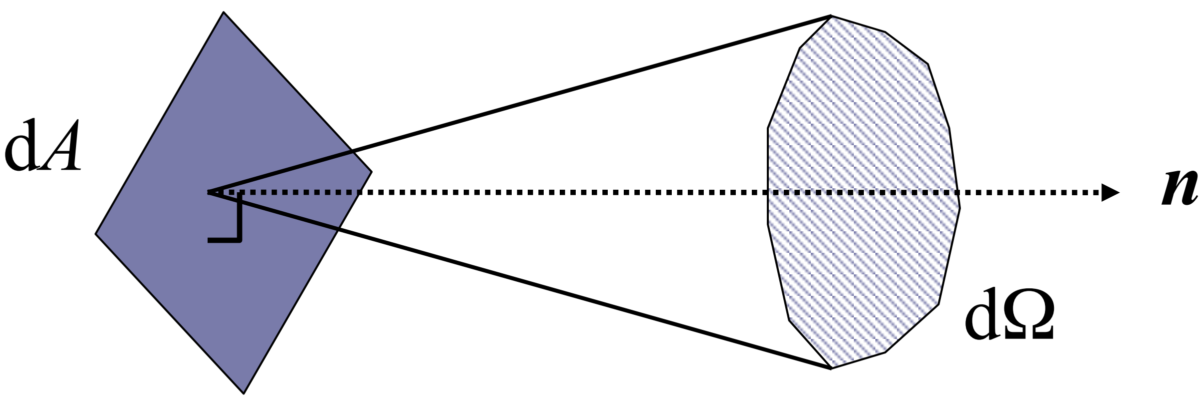


Intracluster medium (Sunyaev-Zeldovich effect)
AGN jet, accretion disk

2. Radiation Transfer Equation

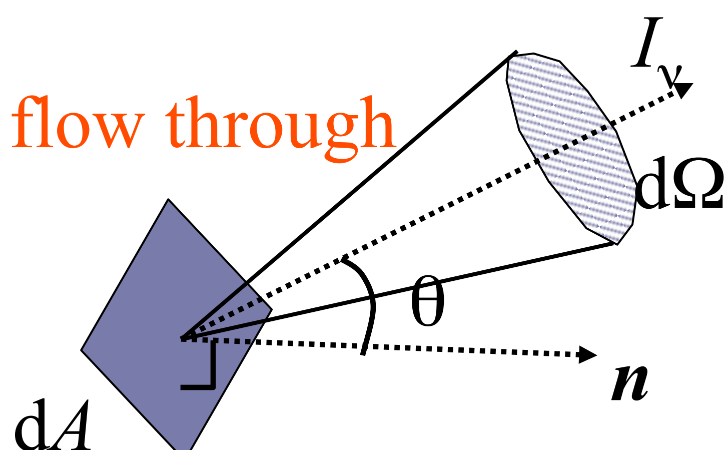
Definition of Intensity (energy flow on a line)

$\mathrm{d}E = I_\nu \, \mathrm{d}A \, \mathrm{d}t \, \mathrm{d}\Omega \, \mathrm{d}\nu$ $\qquad I_\nu$ [erg/cm²/s/sr/Hz]

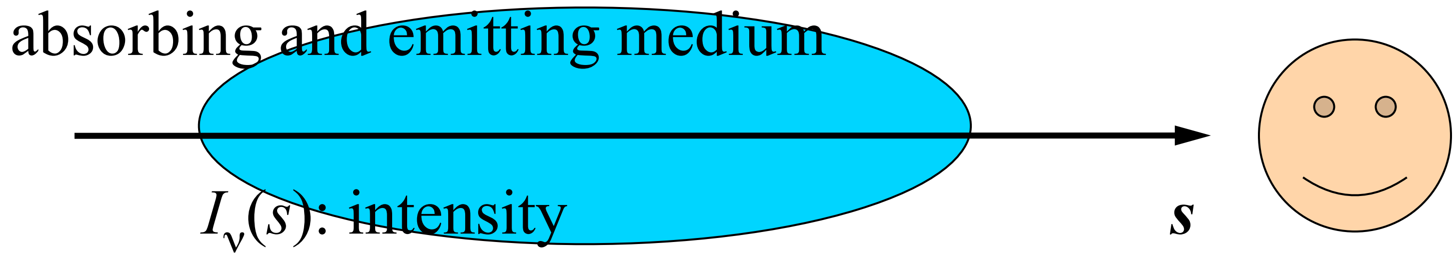


Definition of Flux (net energy flow through a surface) F_ν [erg/cm²/s/Hz]

$$F_\nu = \oint I_\nu \cos\theta \, \mathrm{d}\Omega$$



What is Radiation Transfer Equation?



$$\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = -\kappa_\nu \rho I_\nu + j_\nu$$

Emission

j_ν [erg/cm^3/s/Hz]

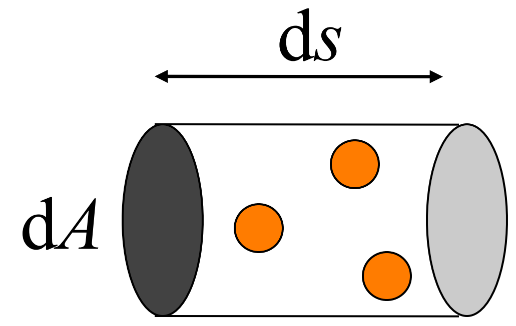
Absorption

κ_ν [cm^2/g]: mass absorption coefficient

ρ [g/cm^3]: density

Homework

Absorption



Total absorbing area: $\sigma_\nu(n\, dA\, ds)$

n: number density of the absorber

σ_ν: absorption cross section of the absorber particle

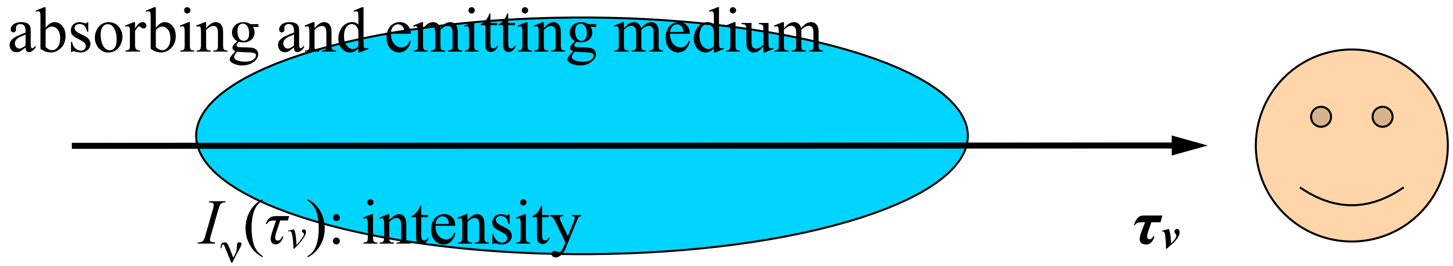
$\Rightarrow -dI_\nu\, dA = I_\nu(n\sigma_\nu dA\, ds) \quad \Rightarrow dI_\nu = -n\sigma_\nu I_\nu\, ds$

$n\sigma_\nu = \rho\kappa_\nu$

ρ: mass density ($= mn$; m is the mass of the absorber)

κ_ν: mass absorption coefficient

Optical Depth



$$\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = -\kappa_\nu \rho I_\nu + j_\nu$$

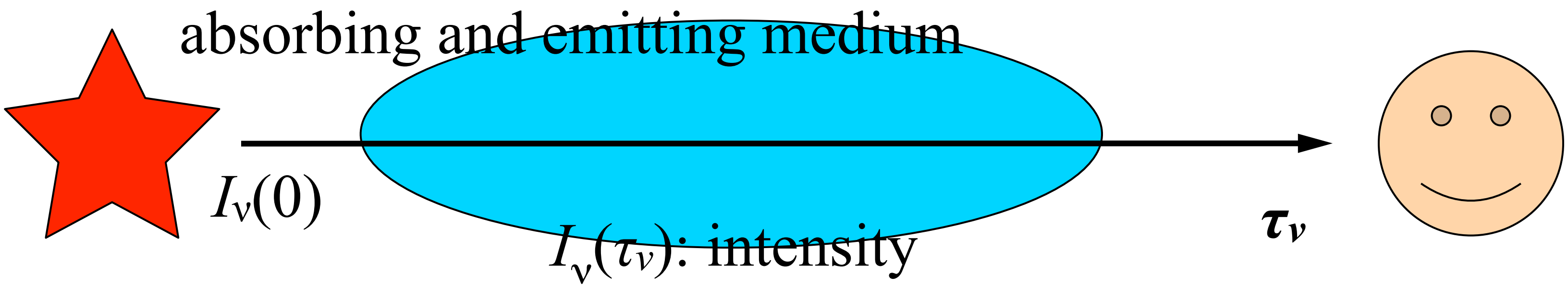
Optical depth: $\mathrm{d}\tau_\nu = \rho\kappa_\nu \mathrm{d}s$ $(= n\sigma_\nu \mathrm{d}s)$

$\propto$ "number of absorptions"

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}\tau_\nu} = -I_\nu + S_\nu$$

$S_\nu = j_\nu/(\rho\kappa_\nu)$: source function

3. Simple Solutions



$$\frac{\mathrm{d}I_\nu}{\mathrm{d}\tau_\nu} = -I_\nu + S_\nu$$

For a constant source function:

$$I_\nu(\tau_\nu) = I_\nu(0)\mathrm{e}^{-\tau_\nu} + S_\nu(1 - \mathrm{e}^{-\tau_\nu})$$

Extinction of bkg light

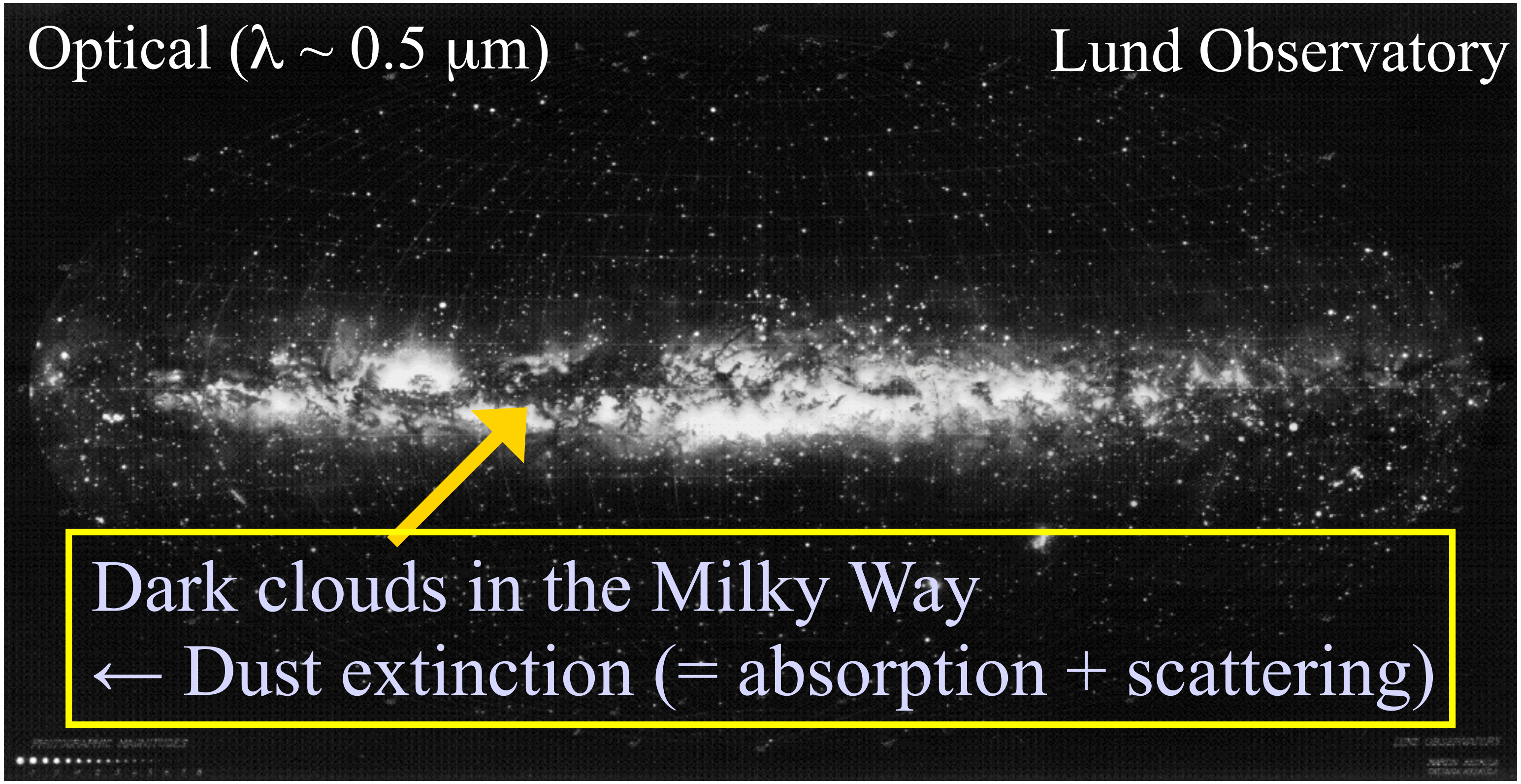
Emission

Extinction as Pure Absorption

With strong background source:

$$I_\nu(\tau_\nu) = I_\nu(0)\mathrm{e}^{-\tau_\nu} + \sout{S_\nu(1 - \mathrm{e}^{-\tau_\nu})}$$

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu}$$



Optically Thick Case

Optically thick: $\tau_\nu >> 1$

$$I_\nu(\tau_\nu) = I_\nu(0)\mathrm{e}^{-\tau_\nu} + S_\nu(1 - \mathrm{e}^{-\tau_\nu})$$

$$I_\nu = S_\nu$$

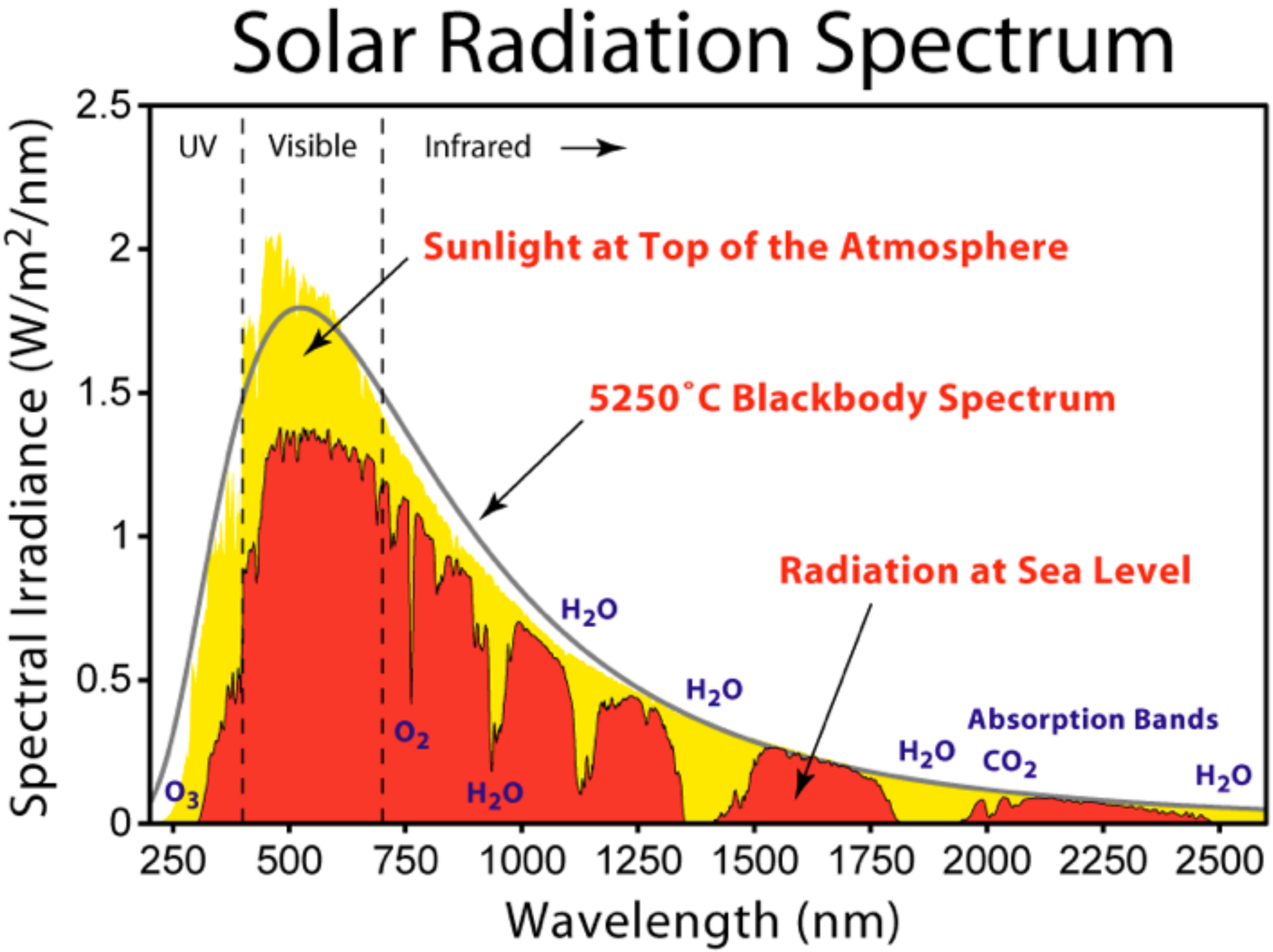
If the material is in **thermal equilibrium**, the resulting intensity should be the Planck function:

Local Thermal Equilibrium (LTE): $S_\nu(\tau_\nu) = B_\nu(T(\tau_\nu))$

($j_\nu = \rho\kappa_\nu B_\nu$): Kirchhoff's law.

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{\exp(h\nu/kT) - 1}$$

Example of Optically Thick Radiation



Optically Thin Emission

Optically thin: $\tau_\nu << 1$

Without background light $$I_\nu(\tau_\nu) = S_\nu(1 - e^{-\tau_\nu})$$

$$I_\nu(\tau_\nu) = \tau_\nu S_\nu$$

column density

For LTE $$I_\nu(\tau_\nu) = \tau_\nu B_\nu = \kappa_\nu \rho L B_\nu(T)$$

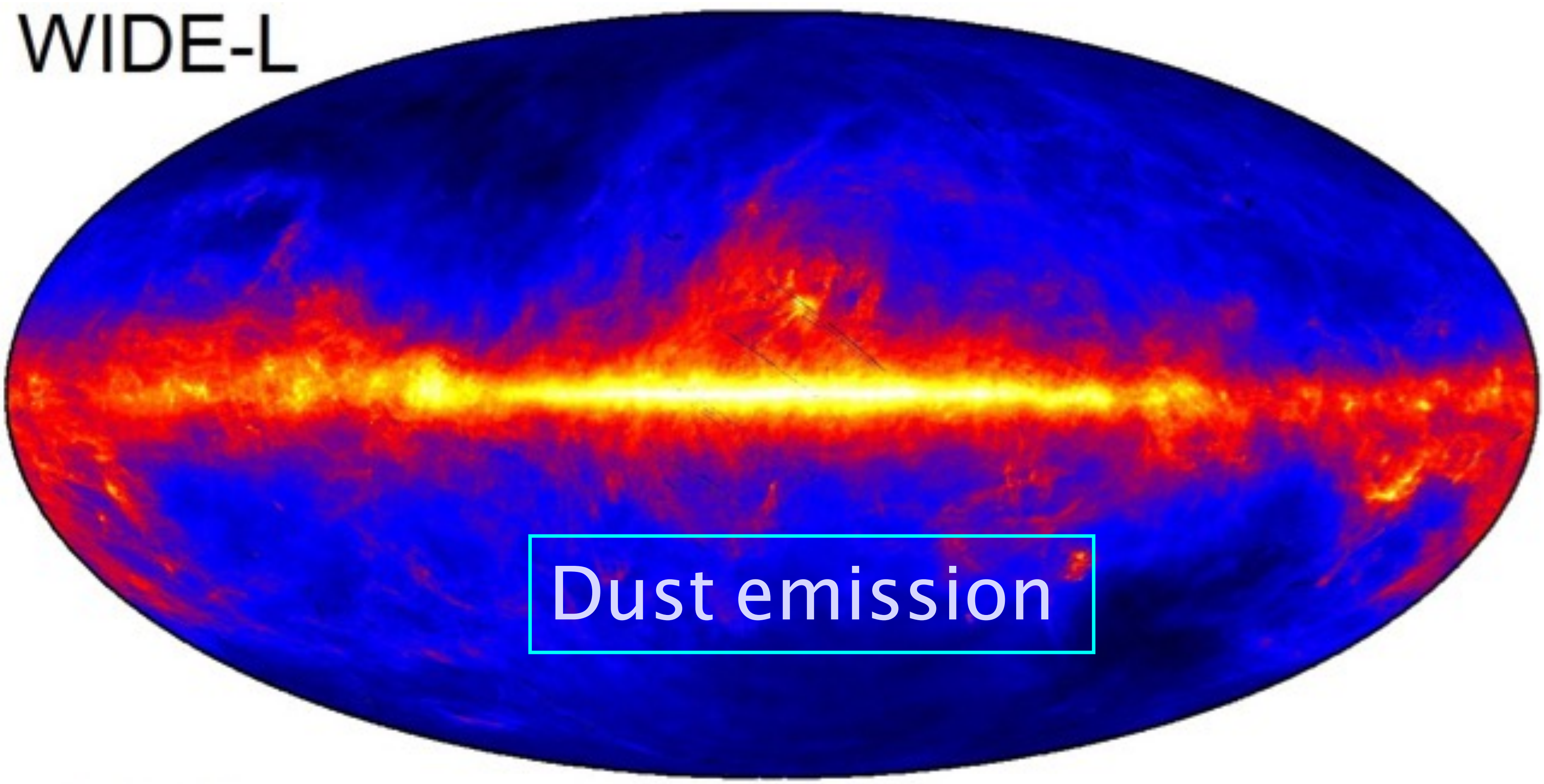
Optically thin radiation reflects:

(1) column density (ρL)

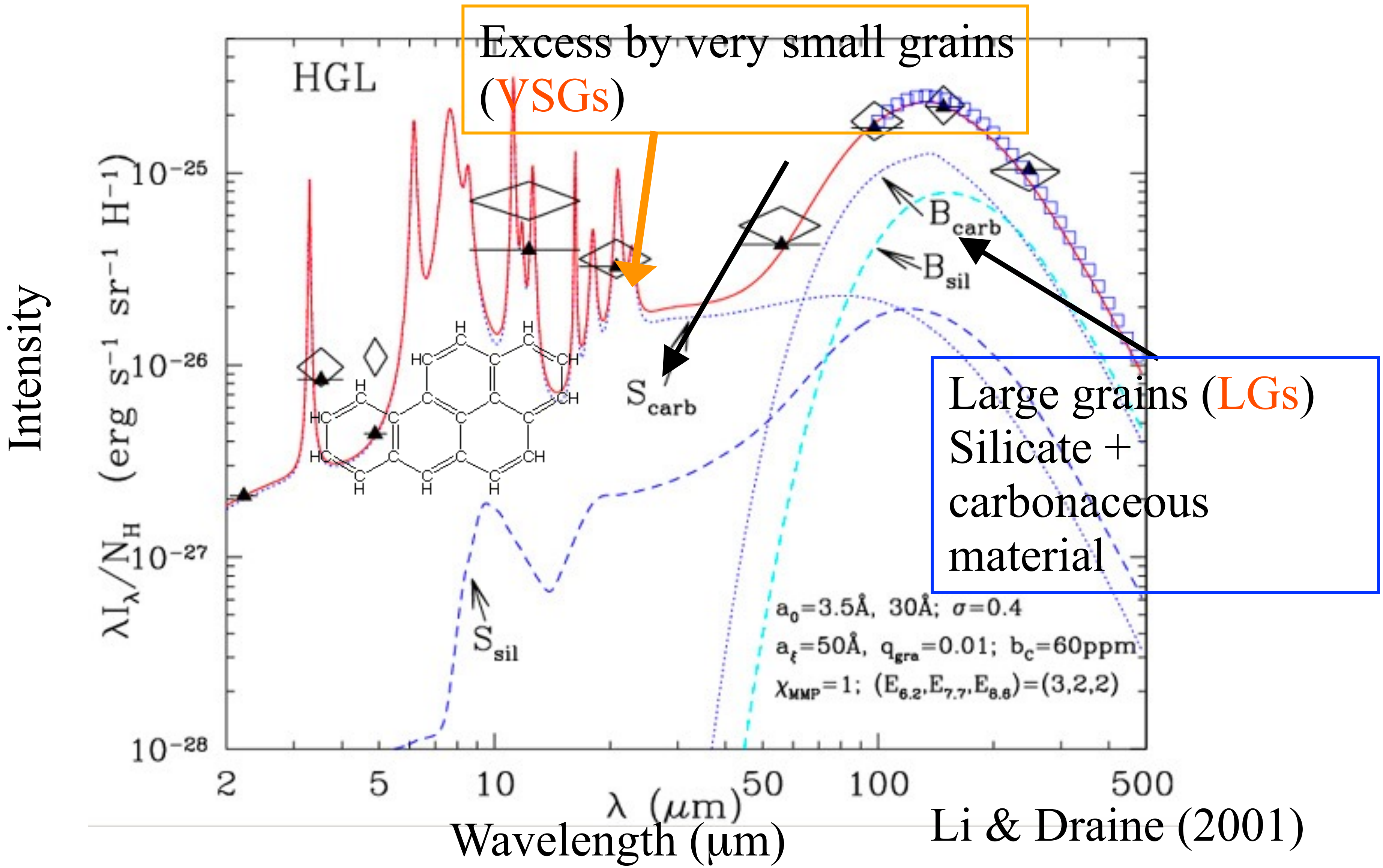
(2) optical properties (κ_ν)

Milky Way in the Far Infrared

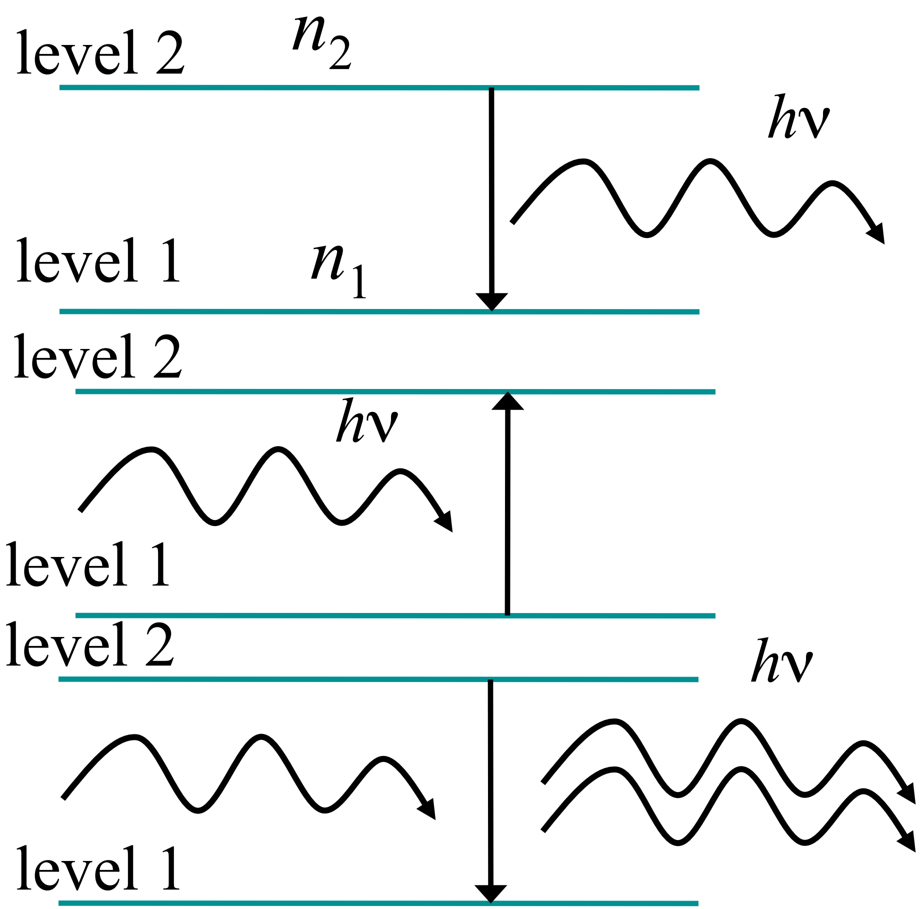
AKARI 140 μm



Optical Properties of Dust



4. The Einstein Coefficients



Spontaneous emission

A_{21} (transition probability per unit time)

Absorption

B_{12}J (transition probability per unit time)

Stimulated emission

B_{21}J (transition probability per unit time)

$$\mathrm{J} = \int_0^\infty J_\nu \phi(\nu) d\nu$$

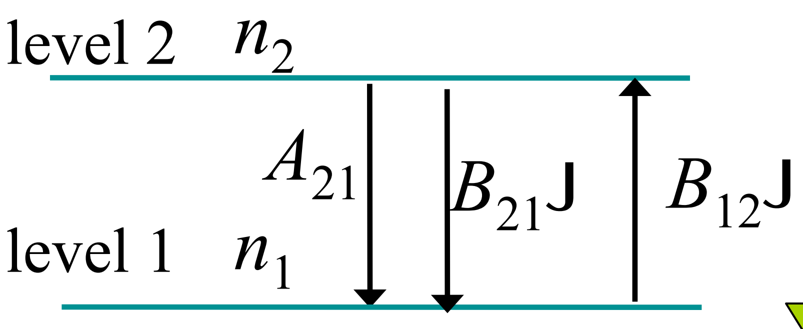
$\phi(\nu)$: line profile function

$$\int_0^\infty \phi(\nu) d\nu = 1$$

Einstein Relations

Thermodynamic equilibrium

$$n_1 B_{12} J = n_2 A_{21} + n_2 B_{21} J \quad \Rightarrow \quad J = \frac{A_{21}/B_{21}}{(n_1/n_2)(B_{12}/B_{21}) - 1}$$



$$\frac{n_1}{n_2} = \frac{g_1}{g_2} \exp(h\nu/kT)$$

$$J = \frac{A_{21}/B_{21}}{(g_1 B_{12}/g_2 B_{21}) \exp(h\nu/kT) - 1}$$

should be the Planck function.

$$g_1 B_{12} = g_2 B_{21}$$

$$A_{21} = (2h\nu^3/c^2) B_{21}$$

Absorption and Emission Coefficients

$$j_\nu = \frac{h\nu}{4\pi} n_2 A_{21} \phi(\nu) \qquad \alpha_\nu = \frac{h\nu}{4\pi} \phi(\nu)(n_1 B_{12} - n_2 B_{21})$$

$$S_\nu = \frac{n_2 A_{21}}{n_1 B_{12} - n_2 B_{21}} \qquad (\alpha_\nu = \rho \kappa_\nu)$$

Using the Einstein relations,

$$\alpha_\nu = \frac{h\nu}{4\pi} n_1 B_{12} (1 - g_1 n_2 / g_2 n_1) \phi(\nu)$$

$$S_\nu = \frac{2h\nu^3}{c^2} \left(\frac{g_2 n_1}{g_1 n_2} - 1 \right)^{-1}$$

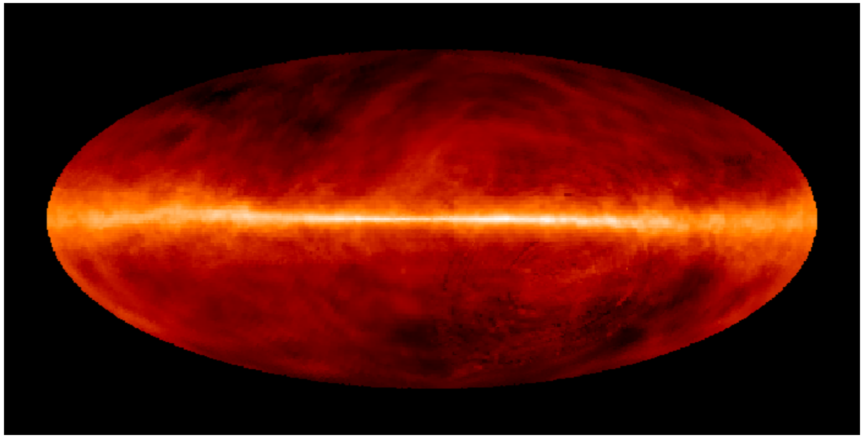
Correction for the stimulated emission

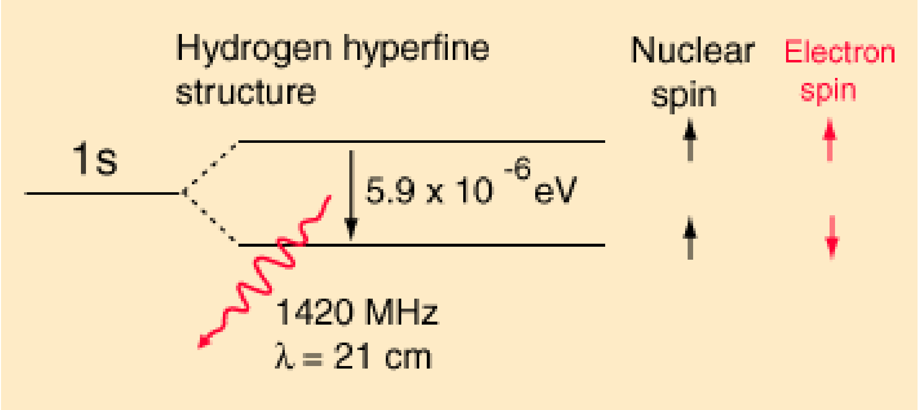
For LTE:

$$\alpha_\nu = \frac{h\nu}{4\pi} n_1 B_{12} \left[1 - \exp(-h\nu / k_{\rm B} T) \right] \phi(\nu)$$

$$S_\nu = B_\nu(T)$$

Example for the Correction for the Stimulated Emission: Atomic Hydrogen (21 cm)





$$h\nu/k_\mathrm{B}T << 1 \rightarrow \quad \alpha_\nu = \frac{h\nu}{4\pi} n_1 B_{12} \frac{h\nu}{k_\mathrm{B}T}\phi(\nu) \equiv K_0 n_1 \frac{h\nu}{k_\mathrm{B}T}\phi(\nu)$$

$$\tau_\nu = K_0 \frac{N_\mathrm{HI}}{4} \frac{h\nu}{k_\mathrm{B}T}\phi(\nu) \qquad (g_2 = 3,\ g_1 = 1)$$

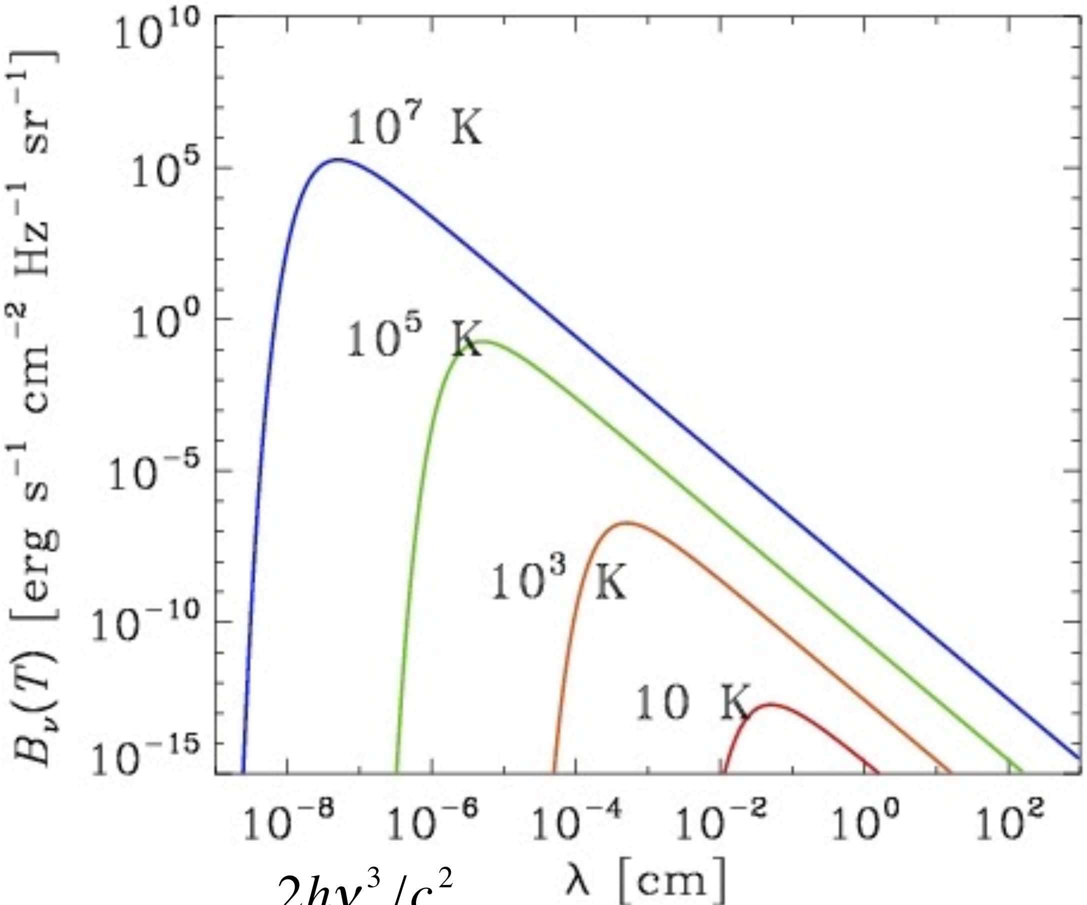
Rayleigh-Jeans Approx. →

$$I_\nu = \tau_\nu B_\nu(T) \simeq \frac{2h\nu^3}{c^2} K_0 \frac{N_\mathrm{HI}}{4}\phi(\nu)$$

known constant →

$$I = \int I_\nu \, \mathrm{d}\nu \simeq \frac{2h\nu^3}{c^2} K_0 \frac{N_\mathrm{HI}}{4}$$

Appendix: The Properties of $B_\nu(T)$



$$B_\nu(T) = \frac{2h\nu^3/c^2}{\exp(h\nu/kT) - 1}$$

$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{\exp(hc/k\lambda T) - 1}$$

(1) Monotonic increase with T.

(2) Peak ($\partial B_\lambda/\partial\lambda|_{\lambda = \lambda max} = 0$): $\lambda_{max} T = 0.29$ cm deg (Wien displacement law).

(3) $h\nu << kT \Rightarrow B_\nu(T) \sim (2\nu^2/c^2)kT$ (Rayleigh-Jeans law)

(4) $\int_0^\infty \pi B_\nu(T) d\nu = \sigma T^4$

$\sigma = 2\pi^5 k^4/(15c^2h^3)$
Stefan-Boltzmann constant

Further Reading

- Rybicki, G. B., & Lightman, A. P. 1979, "Radiative Processes in Astrophysics" (Wiley: New York)