Modeling convection in planetary interiors: application to large icy moons

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with …

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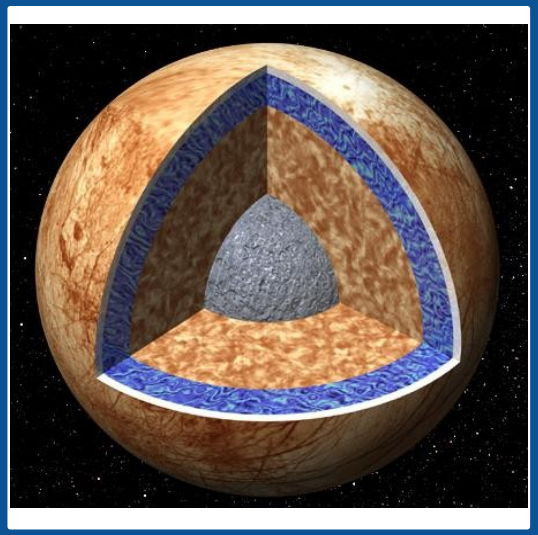

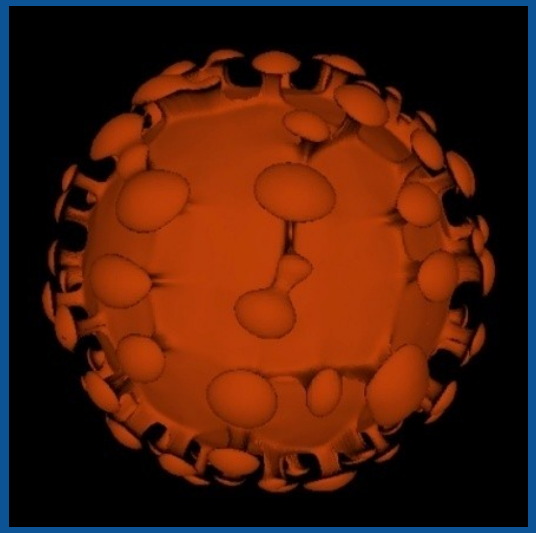

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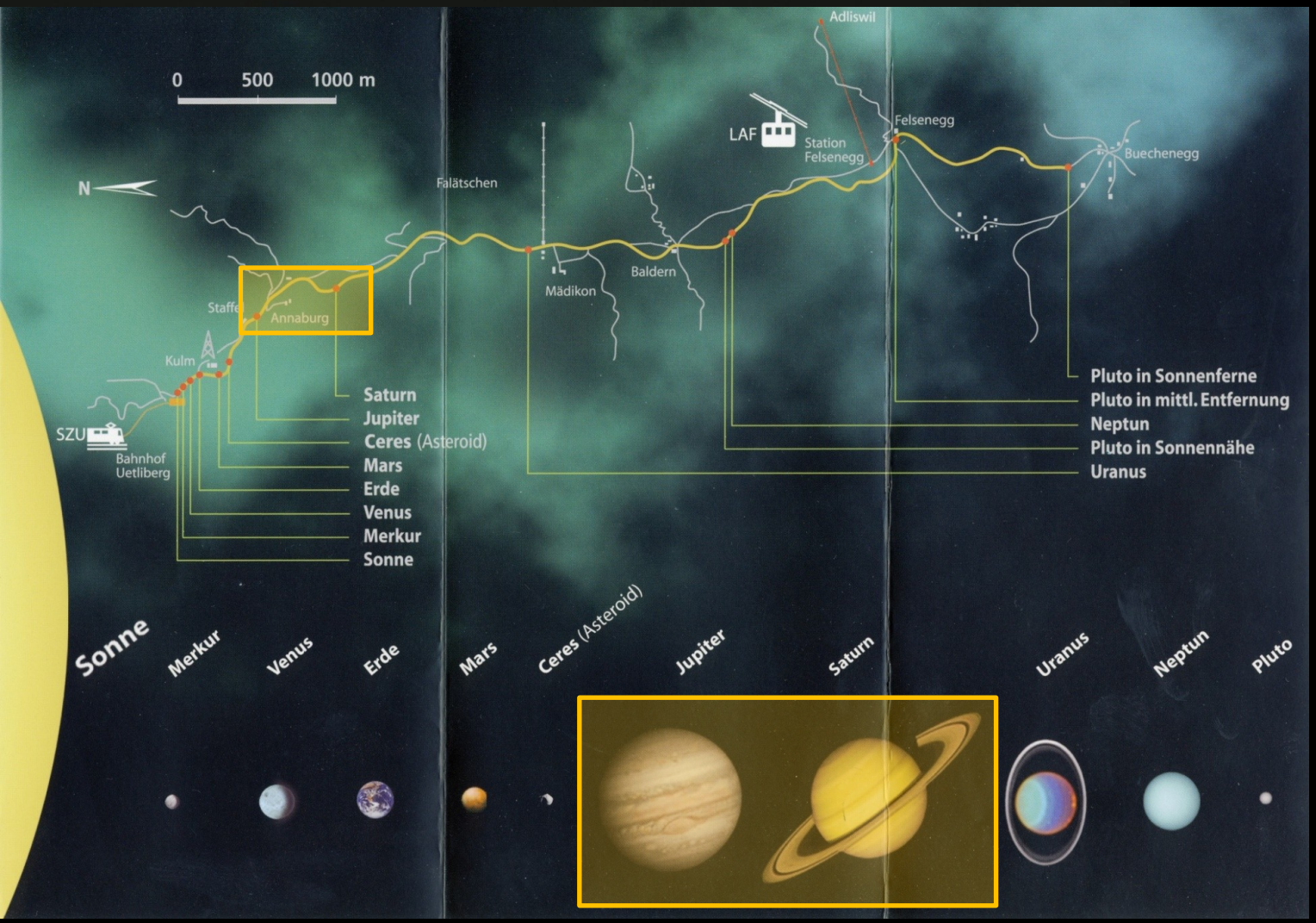
"The Earth is not just an ordinary planet ! ..."

''... One can count there, 111 kings (not forgetting, to be sure, the Negro kings among them), 7,000 geographers, 900,000 business men, 7,500,000 tiplers 311,000,000 conceited men – that is to say, about 2,000,000,000 grow-ups''

(Antoine de Saint-Exupery, The Little Prince)

And it is a dynamic planet (Plate tectonics!)



Icy moons interiors also have dynamics.

Can be modeled with similar tools are for the Earth

How to maintain sub-surface ocean in icy moons interior?

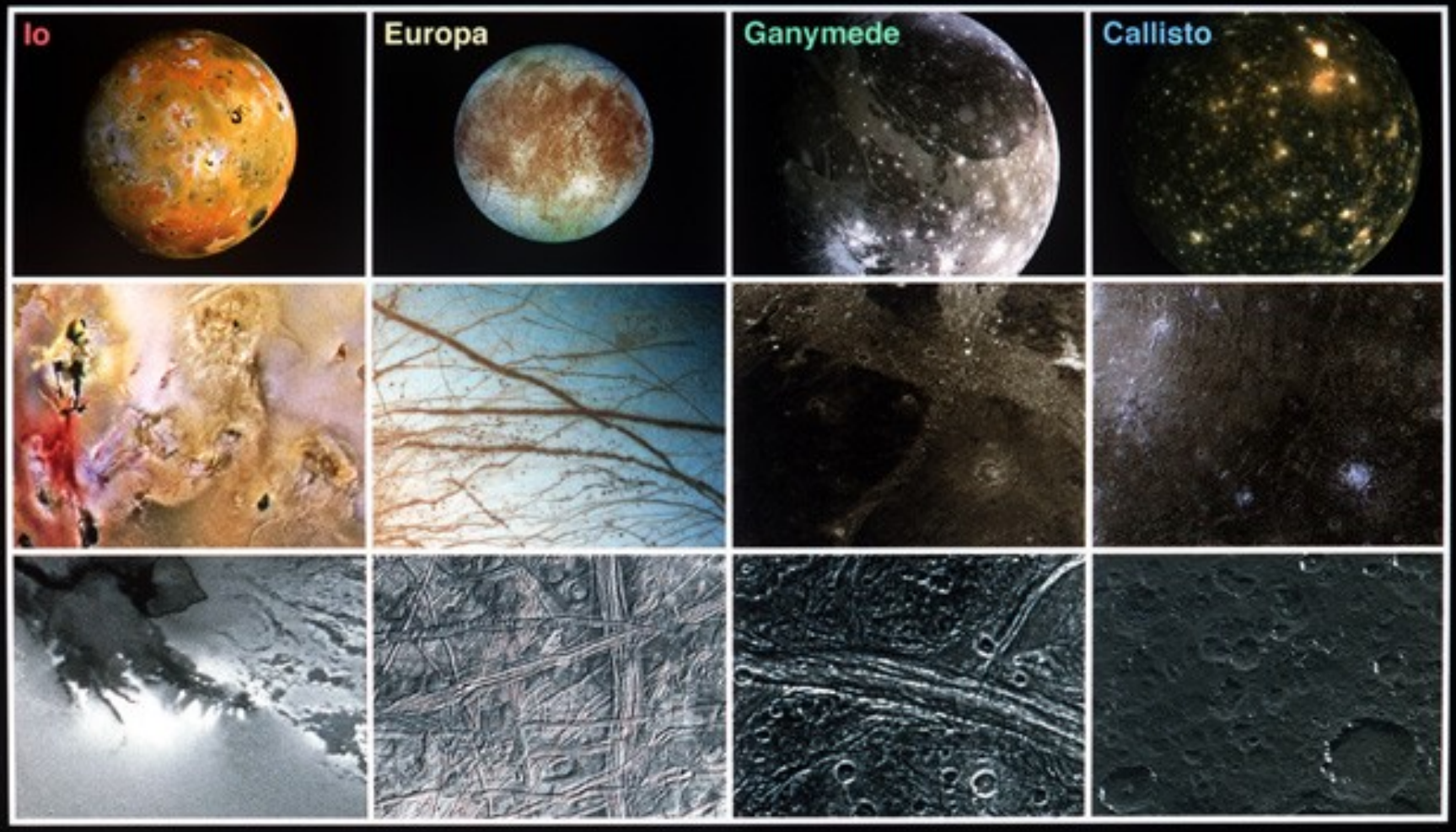
Icy moons: observations and properties

Modeling thermal convection in planetary interiors

The role of anti-freeze compounds in maintaining sub-surface ocean in icy moons interior

Icy moons: observations and properties

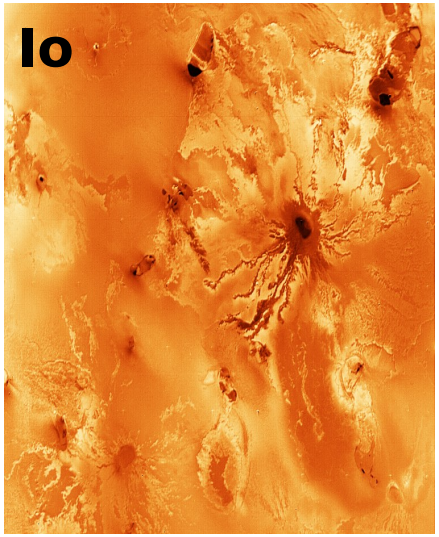
Galilean satellites: surface observations

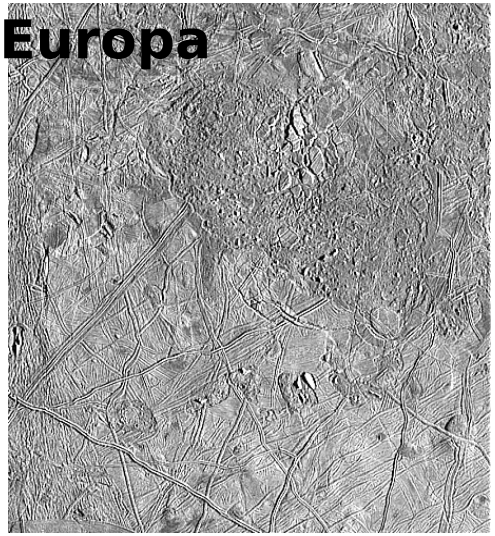


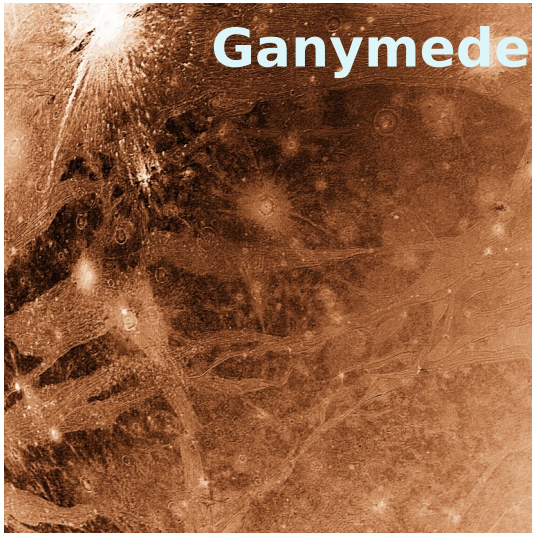
Large differences in surface composition and surface geology

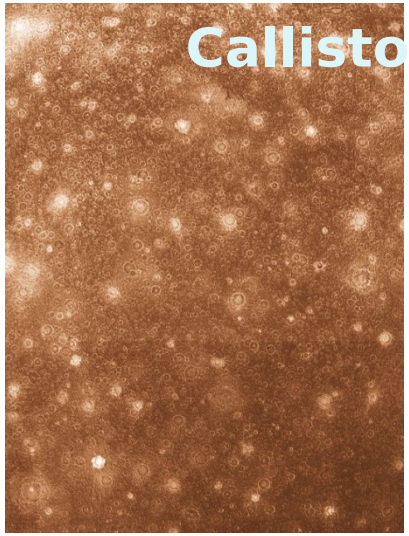
Galilean satellites: surface observations

- Io: sulphur and silicates; active volcanism + constant resurfacing; interior partially molten (tidal heating).
- Europa: ices; young surface (few craters), few topography, but many cracks and fault indicating tectonic activity.
- Ganymede: ices, two type of terrain: dark old (~ 3Gyr) areas, and light grooved younger areas experiencing tectonic activity.
- Callisto: ices, very old surface (highly craterized), no visible cryo-volcanoes, cracks.





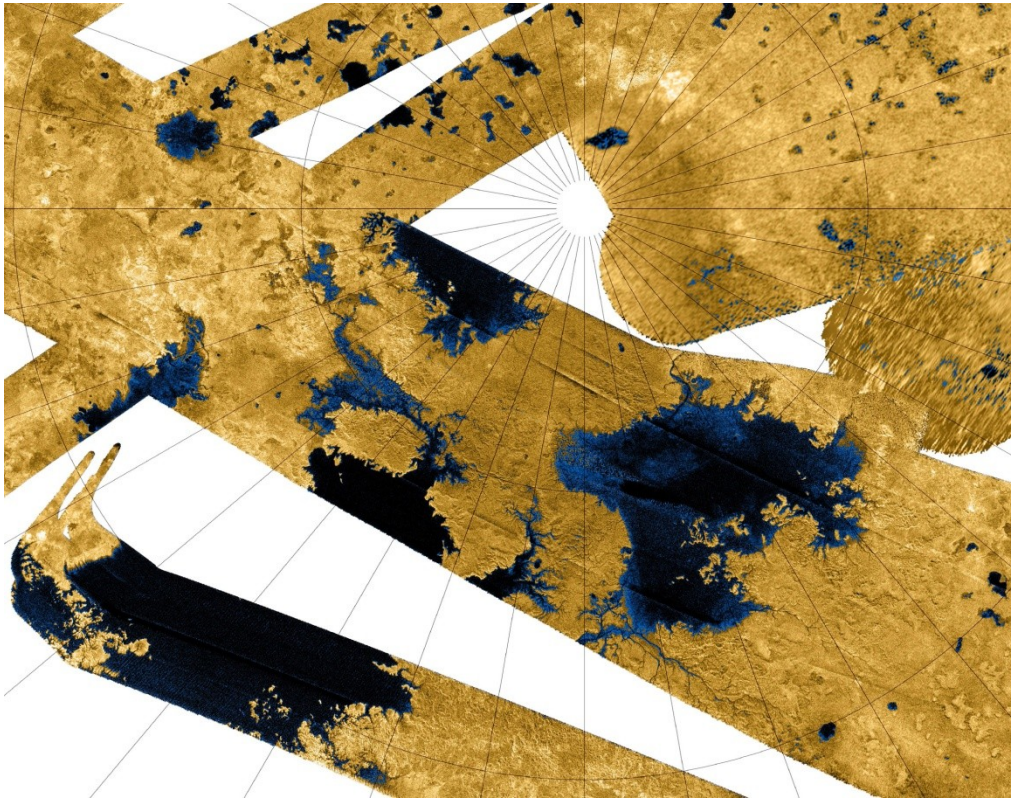


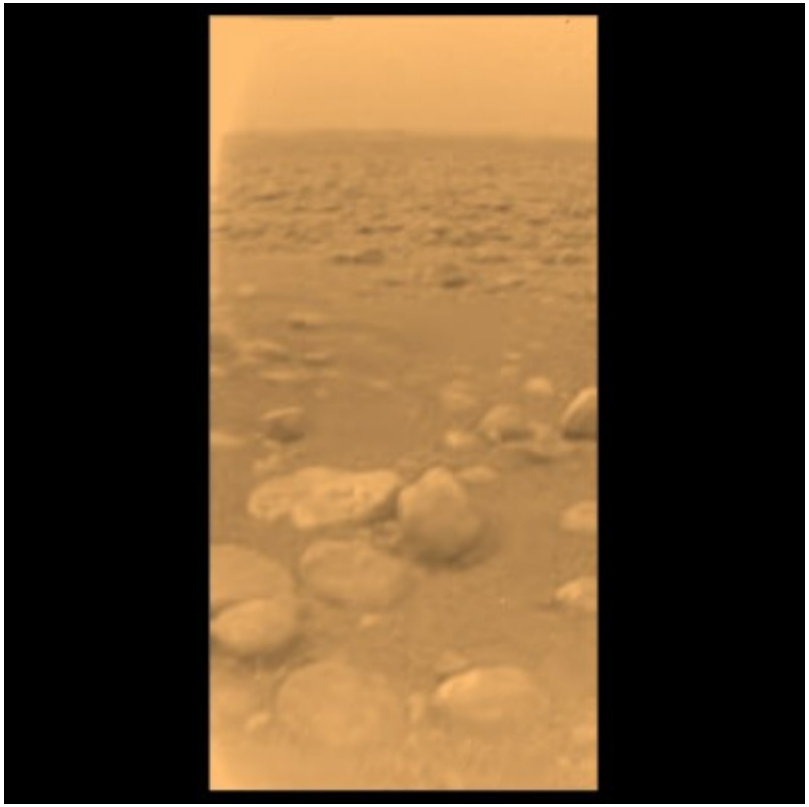


Titan : surface observations

- Covered by thick (~ 1.5 atm) atmosphere, mainly N2 and CH4.

- Cassini-Huygens: rocks + icy surface. Evidence for methane/ethane lakes (from reflectivity data). Altimetry radar suggest moderate topography (few hundred meters) + craters.





Density and moment of inertia

Planet/moon	***Radius*** (km)	***Mass*** (1020 kg)	***Density*** (g/cm3)	***J***
Io	1821	893	3.53	0.370(1)
Europa	1565	480	3.02	0.347(1)
Ganymede	2634	1482	1.94	0.311(1)
Callisto	2403	1076	1.85	0.358(1)
Titan	2575	1346	1.88	0.340(2)
Triton	1353	215	2.07	-
Pluto	1152	131	2.05	-
Charon	604	15	1.65	-

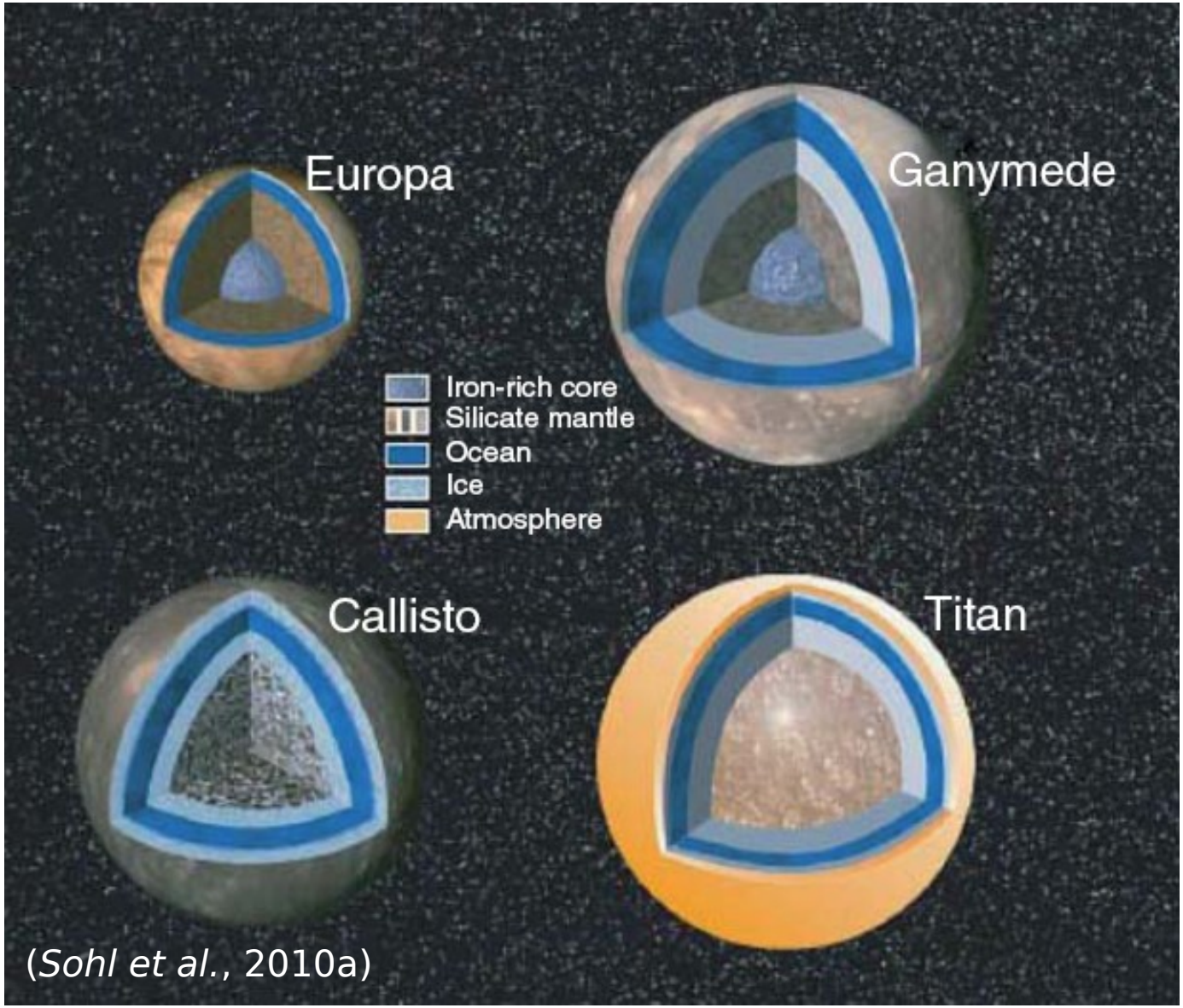
(1) *Showman and Malhotra* (1999); (2) Iess et al. (2010).

- Densitie … cks (Io) and ice +
- Moment

$$J = \frac{C}{MR^2}$$

Io, Europa, and Ganymede are differentiated. Callisto and Titan are partially differentiated (mantle is a mix of ice and rocks).

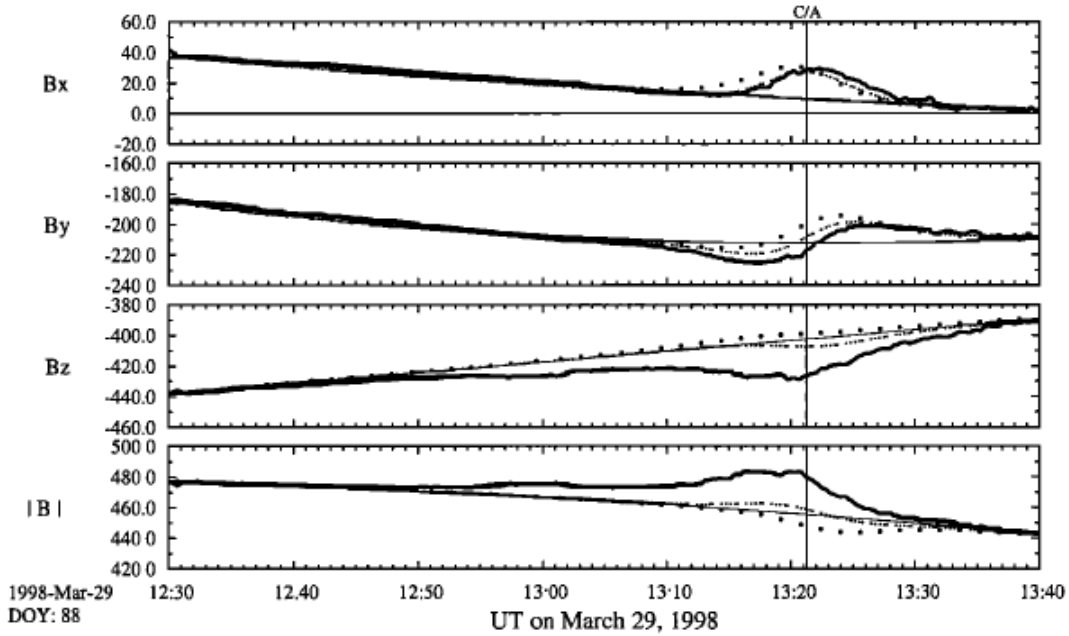
Possible internal structure



(*Sohl et al.*, 2010a)

Induced magnetic field

- Induced magnetic fields (~ 100 nT) observed during Galileo flybys at Europa, Ganymede, and Callisto (*Khurana et al.*, 1998; *Kivelson et al.*, 2002)

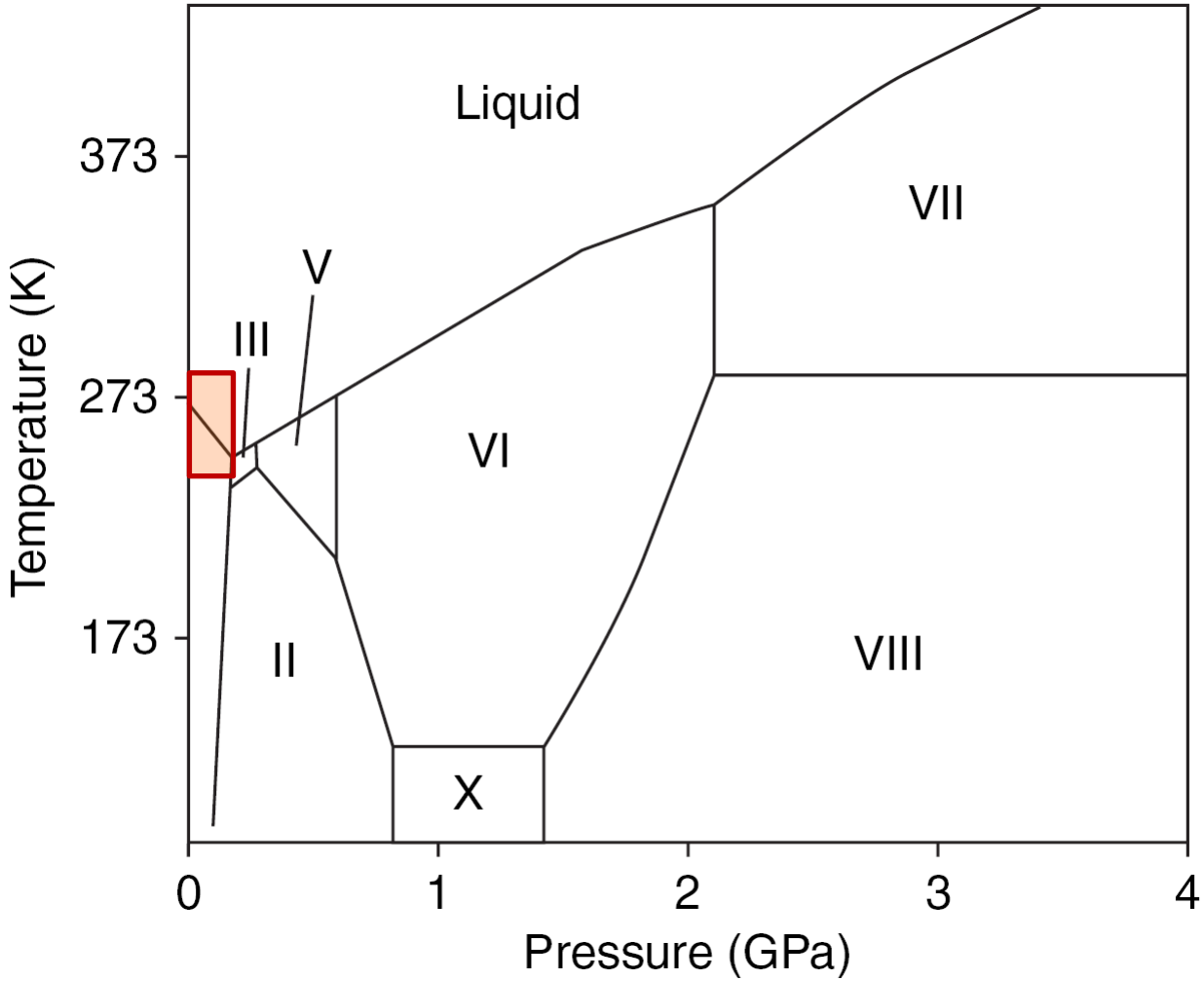


X	-9.84	-8.17	-6.56	-4 95	-3.31	-1 65	0.05	1.75
Y	8.06	6 16	4.34	2.51	0.67	-1 16	-2.96	-4.72
Z	0.29	0 32	0.34	0.37	0.40	0.42	0.43	0.43
R	12.72	10 24	7.87	5.56	3.39	2.06	2.99	5.06

Suggests the presence of liquid layer (water + electrolytes) around 100-200 km below the surface

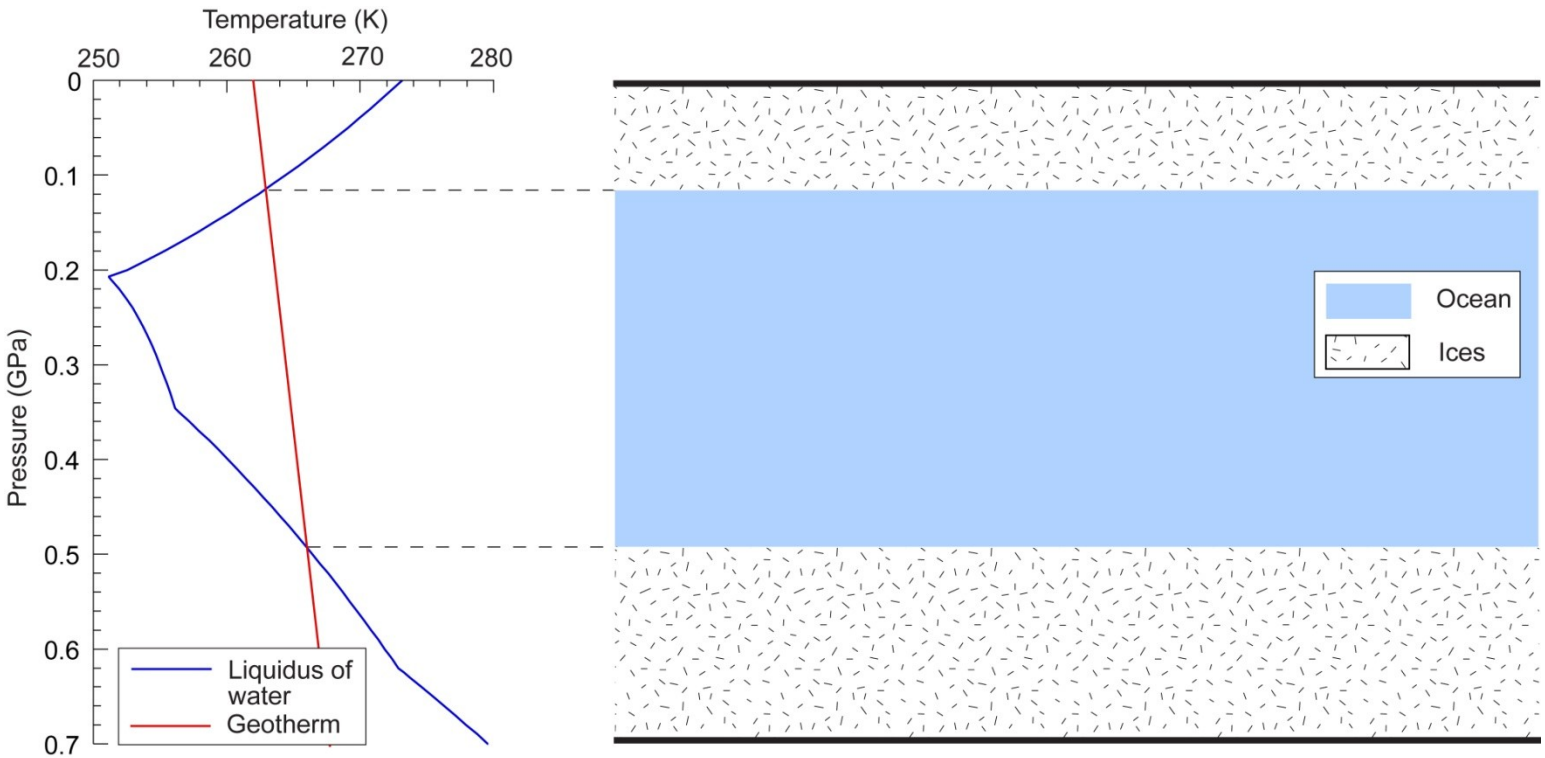
Water phase diagram

Solid/liquid phase transition endothermic up to 0.21 GPa



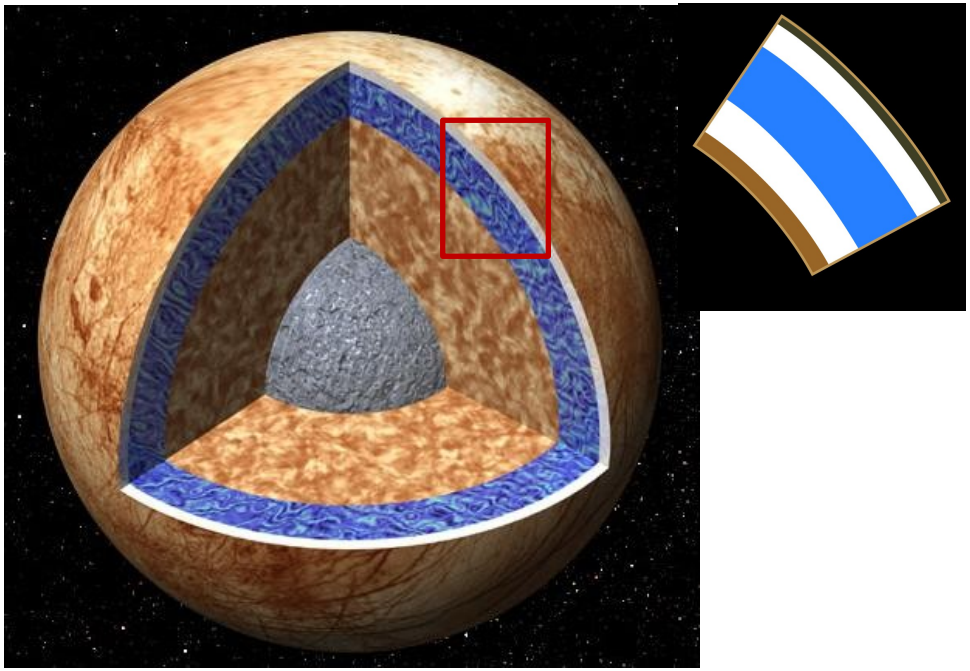
Cristallyzation of the primodial ocean of icy moons

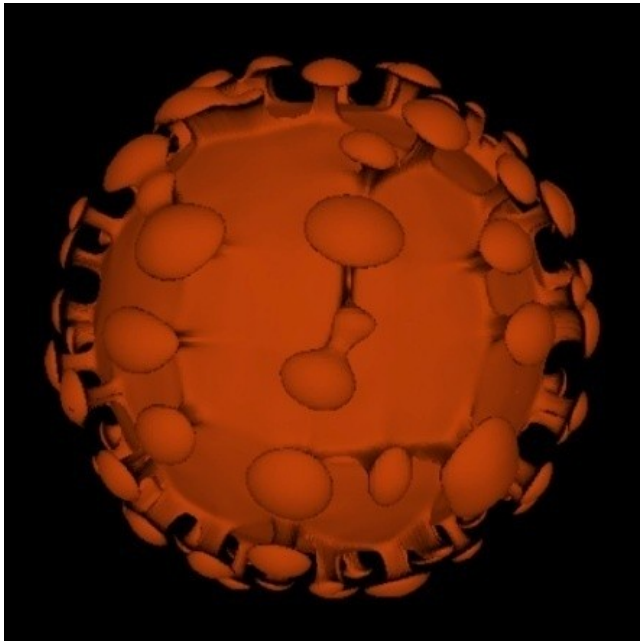
- The primordial ocean of icy moons of giant planets crystallizes simultaneously at its top and at its bottom *(e.g., Lewis, 1973)*:



Icy satellites: structure and dynamics

- Heat transfer through the outer Ice I layer controls the cooling of the body, in particular the crystallization of the ocean.

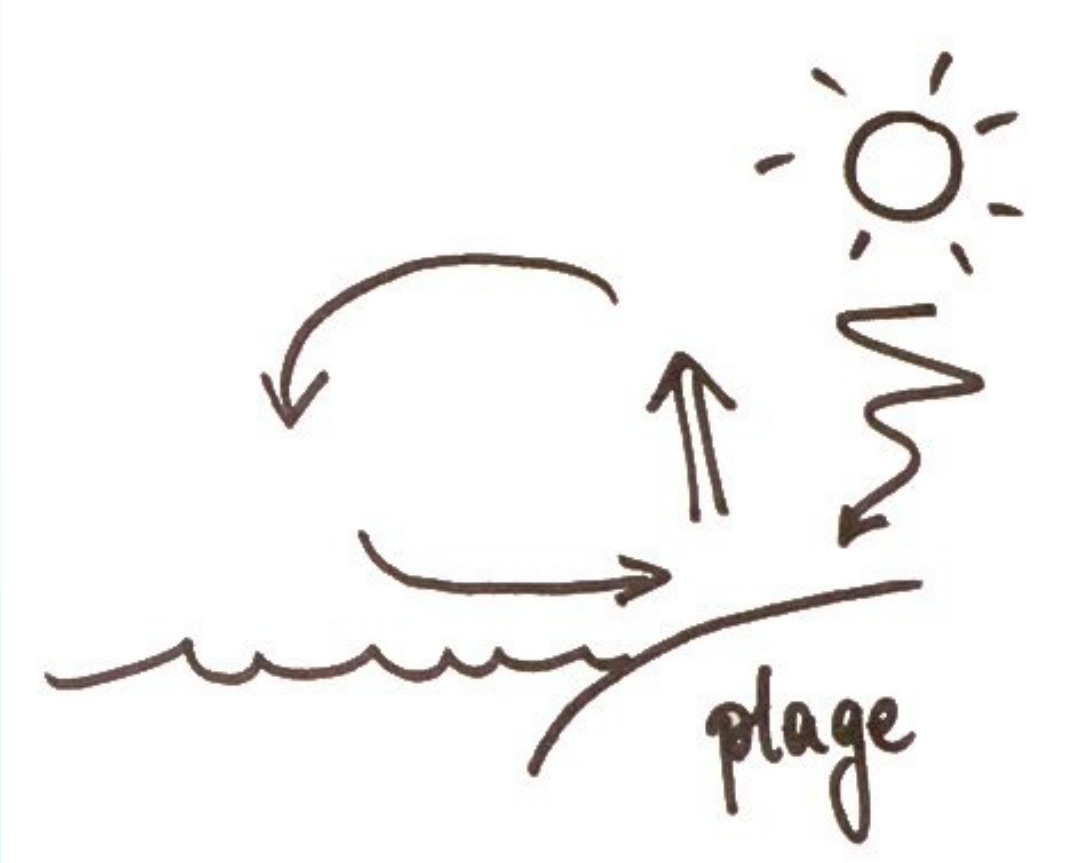




- If it is thick enough, the outer ice I layer may convect.

- If convection in the outer ice layer is not efficient enough, crystallization stops and a sub-surface salty water ocean can be maintained *(e.g., Deschamps and Sotin, 2001)*.

Modeling convection in planetary interiors



principles

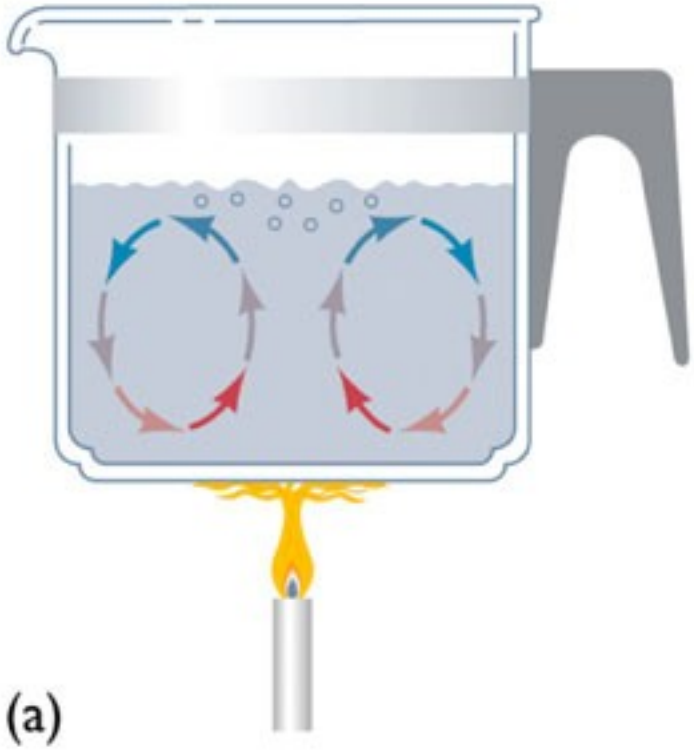
- Thermal convection is a mode of heat transfert through advection of mass.

- Motor: lateral variations in density *(e.g., generated by lateral variations in temperature)*, inducing a global circulation of the fluid.

- Several factors are opposing this flow:
  - *Viscous forces.*
  - *Diffusion of heat.*

- Convection settles if the buoyancy is larger than the viscous forces, and if the fluid conduct heat poorly.

- Depends on the properties of the fluid (viscosity, conductivity, …) and of the system (geometry).



Convection: importants numbers

- Rayleigh number:

$$Ra = \frac{\alpha \rho g \Delta T D^3}{\eta \kappa}$$

- Ratio between buoyancy and viscous forces, measuring the vigor of convection.
- Convection occurs if *Ra* is larger than a critical Rayleigh number, which depends on the fluid (e.g., rheology) and system (e.g., geometry) properties.
- Earth's mantle: *Ra* ~ 106-107.

- Prandtl number:

$$Pr = \frac{\nu}{\kappa}$$

- Ratio of the characteristic times for the diffusion of heat and for the diffusion of momentum.
- Earth's mantle: *Pr* ~ 1025.

- Nombre de Nusselt:

$$Nu = \frac{\Phi_{\text{convec}}}{k \Delta T / D}$$

- Ratio between the horizontally averaged convective heat flux and the conductive heat flux, measuring the efficiency of heat transport.
- Increases with the vigor of convection. Depends on the fluid (e.g., rheology) and system (e.g., geometry) properties.

Scaling laws

- Relate observables to model variables.

- Provide a general expression that can be applied to systems with similar properties, but various dimension, *e.g.*, the thermal evolution of planetary mantles.

- Give some insight on the physical processes at work.

- Example: convective heat transfer *(Nusselt number)* the vigor of convection *(Rayleigh number)*:

$$Nu = aRa^b$$

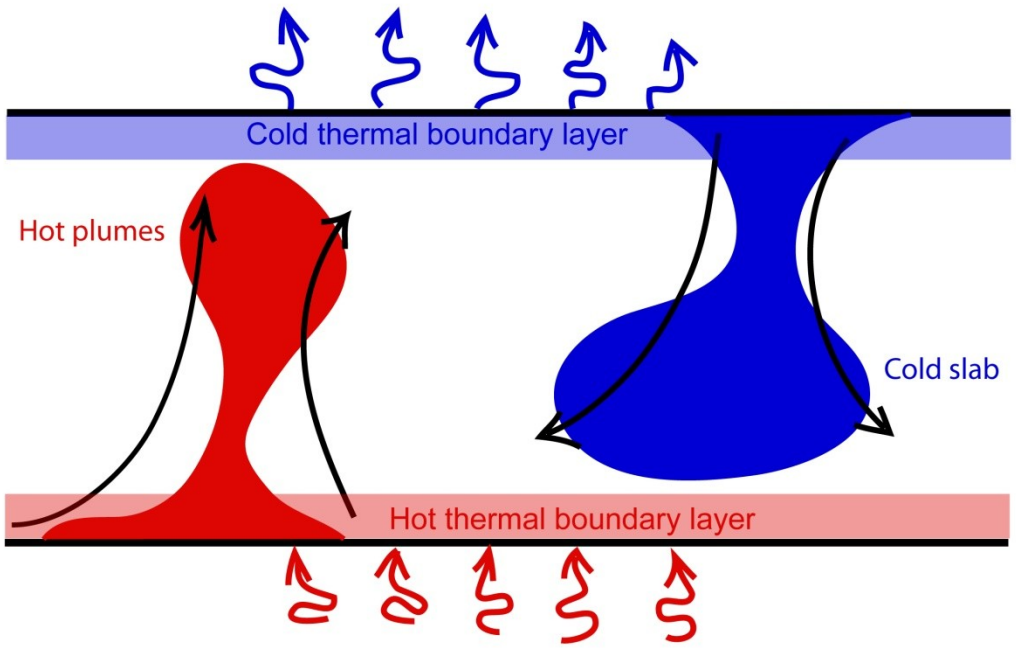
Reference	*a*	*b*
TBL analysis	0.294	1/3

Interaction between plumes/slabs and the opposite TBL.

Convection in planetary mantles

- A simple model: Rayleigh-Bénard convection:

Growth of plumes/slabs controlled by instabilities in the hot/cold thermal boundary layers (TBL).



- Degrees de complexity:

- *Mode of heating (volumetric and/or basal)*
- *Geometry (spherical shell)*
- *Rheology (viscosity variations with temperature, depth, …)*
- *Thermo-chemical convection*
- *…*

Consequences on the dynamics of planetary interiors ?

Equations

The modeling of thermal convection is based on three conservation equations . . .

- *Mass conservation*
- *Momentum conservation*
- *Energy conservation*

. . . and a constitutive equation relating stress and strain

Thermo-chemical convection requires an additional equation for the conservation of composition, and additional terms is the momentum equation.

Stress and strain

- Stress:

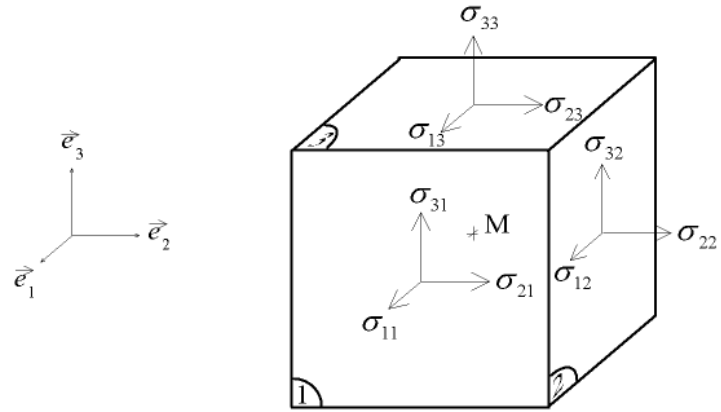
$$\sigma_{ij} = -P\delta_{ij} + \tau_{ij}$$

σij, stress tensor

τij, deviatoric stress tensor

P, hydrostatic pressure

$$P = -\frac{1}{3}\sigma_{kk}$$



- Strain rate:

$$\frac{\partial \varepsilon_{ij}}{\partial t} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

***u**(u1, u2, u3), velocity*

- Newtonian fluid:

$$\sigma_{ij} = -P\delta_{ij} + \eta\left(\underbrace{\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}}_{2\partial\varepsilon_{ij}/\partial t} - \frac{2}{3}\delta_{ij}\underbrace{\frac{\partial u_k}{\partial x_k}}_{\nabla\cdot\mathbf{u}}\right) - \frac{2}{3}\eta_V\delta_{ij}\underbrace{\frac{\partial u_k}{\partial x_k}}_{\nabla\cdot\mathbf{u}}$$

η, viscosity

ηv, bulk viscosity (assumed equal to zero)

Conservation of momentum

- General case: $$\rho\left(\frac{\partial \mathbf{u}}{\partial t}+\mathbf{u}\cdot\nabla\mathbf{u}\right)=\nabla\cdot\boldsymbol{\sigma}+\rho\mathbf{F}$$

σij, stress tensor

***u**, velocity*

*The **acceleration** scales as*  *. . . and the **stress divergence** as* 

(D is the characteristic dimension of the system, κ the heat diffusivity)

- Application to planetary mantles:  acceleration can be neglected

  - *Volume force (gravity):* $\mathbf{F}=\rho g\mathbf{e_z}$
  - *Density anomalies:* $\rho(T)=\rho_0\left[1-\alpha\left(T-T_0\right)\right]$
  - *Non-hydrostatic pressure:* $P=P_{nh}-\rho_0 gz$

- Incompressible, isoviscous fluid: $$-\nabla P_{nh}+\eta\nabla^2\mathbf{u}=\alpha\rho_0\delta T g\mathbf{e_z}$$

  non-dimensional form: $$-\nabla\tilde{P}+\eta_0\nabla^2\tilde{\mathbf{u}}=Ra\tilde{T}\mathbf{e_z}$$

- Variable viscosity, newtonian fluid: $$-\frac{\partial P}{\partial x_i}+\frac{\partial}{\partial x_j}\left[\eta\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}-\frac{2}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij}\right)\right]=\rho F_i$$

Conservation of energy

Energy balance for a fluid heated from below, cooled at the surface, and eventually including internal heat sources

- General case: (assuming heat capacity and conduction are constant):

$$\rho C_V\left(\frac{\partial T}{\partial t}+\mathbf{u}\cdot\nabla T\right)+TC_V\left(\frac{\partial \rho}{\partial t}+\mathbf{u}\cdot\nabla\rho\right)=k\nabla^2 T+\Phi+\rho H$$

- Viscous dissipation: $\Phi=\frac{\partial u_i}{\partial x_j}\sigma_{ij}$

- Internal heat generation: ρH *(e.g., radiogenic heating)*

- Incompressible fluid:

$$\rho C_V\frac{\partial T}{\partial t}=\underbrace{\boxed{k\nabla^2 T}}_{\text{conduction}}-\underbrace{\boxed{\rho C_V\mathbf{u}\cdot\nabla T}}_{\text{advection}}+\underbrace{\boxed{\Phi+\rho H}}_{\text{sources}}$$

Non-dimensional form + neglect viscous dissipation:

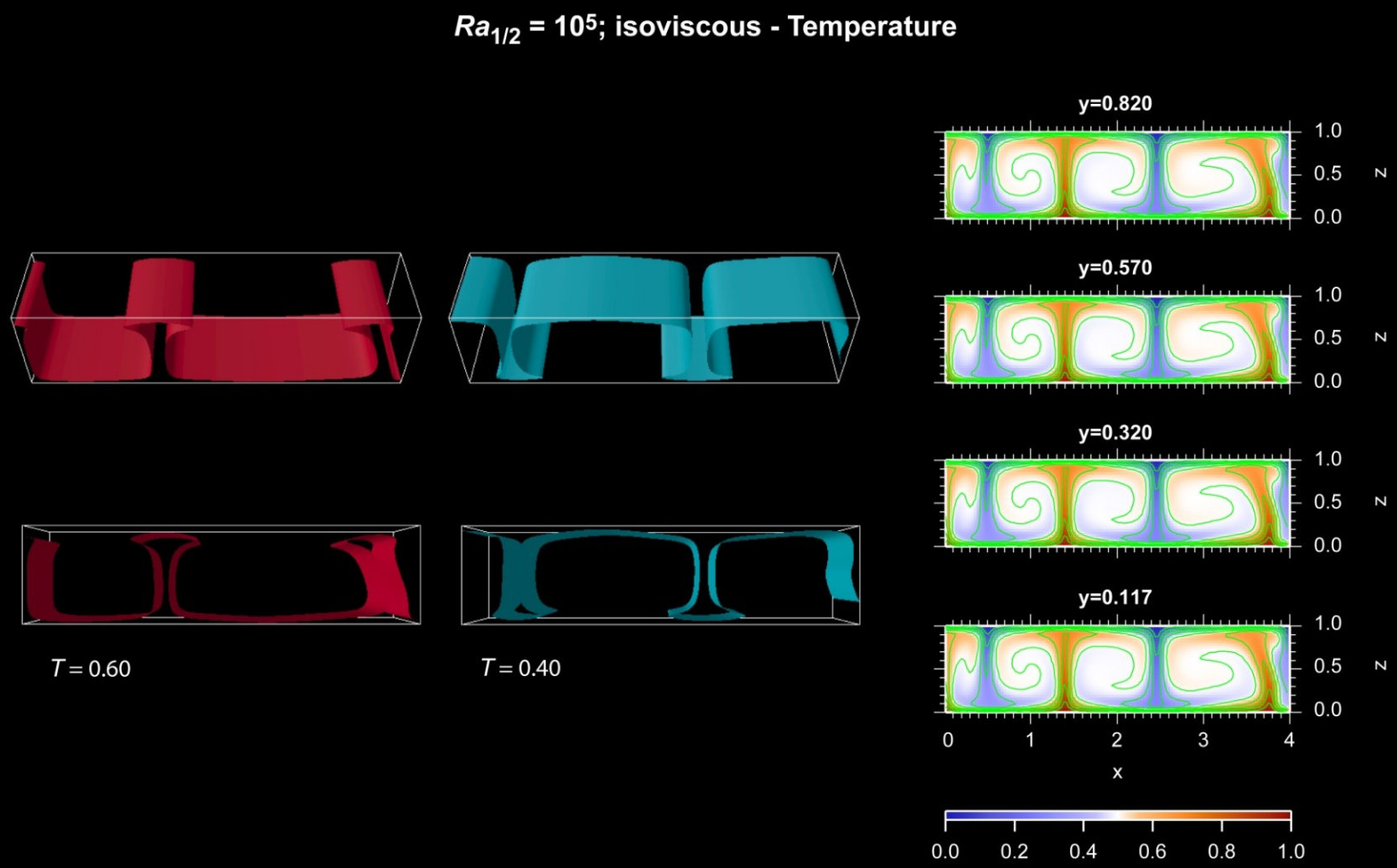
$$\frac{\partial \tilde{T}}{\partial t}=\nabla^2\tilde{T}-\tilde{\mathbf{u}}\cdot\nabla\tilde{T}+h$$

with $\tilde{T}=\frac{T-T_{surf}}{\Delta T}$ $\quad h=\frac{\rho H b^2}{k\Delta T}$

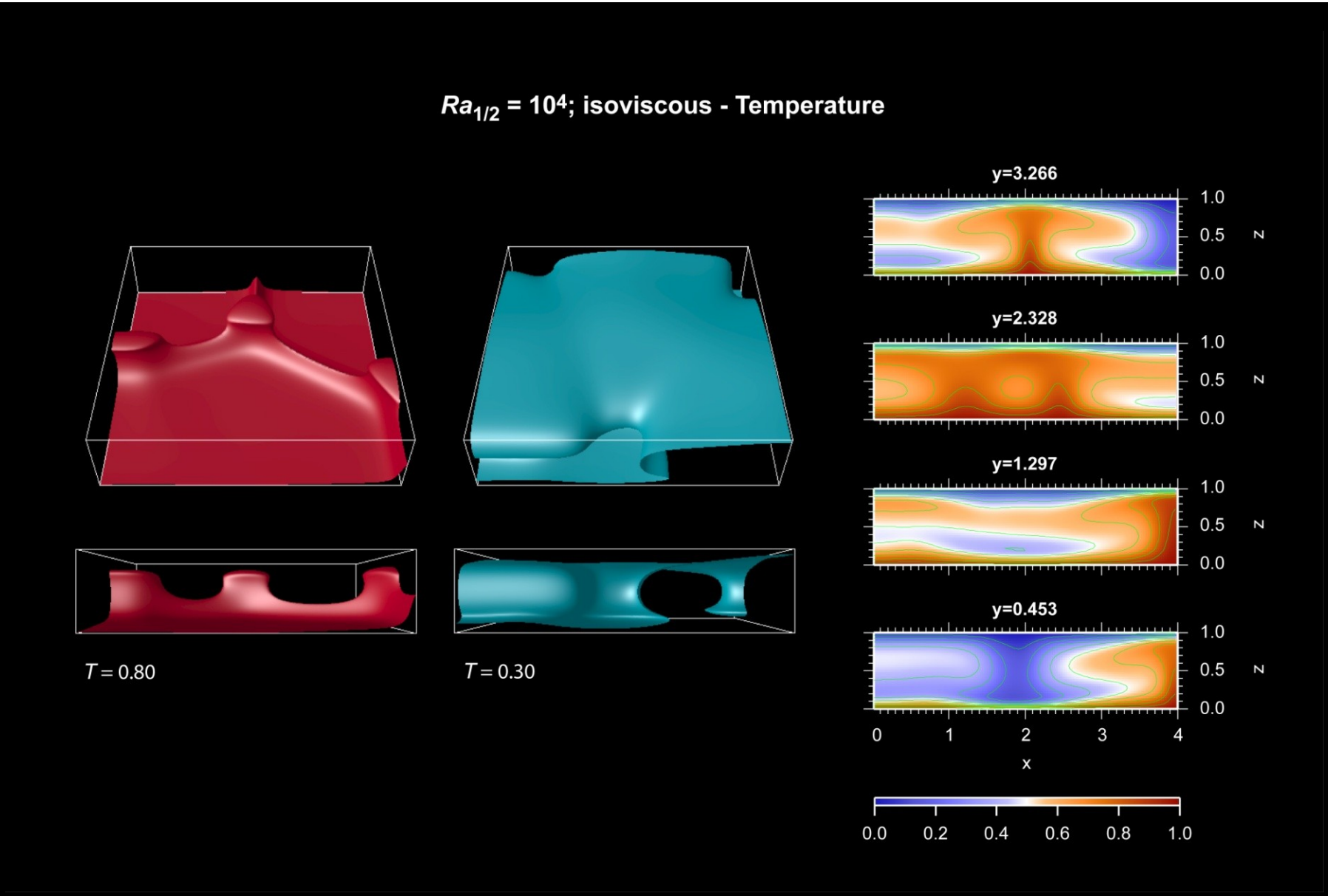
Need super-computing !

***e.g.*: StagYY, spherical grid 512×512×128 nodes runs on 32 processors, ~ few days to week, depending on cases**

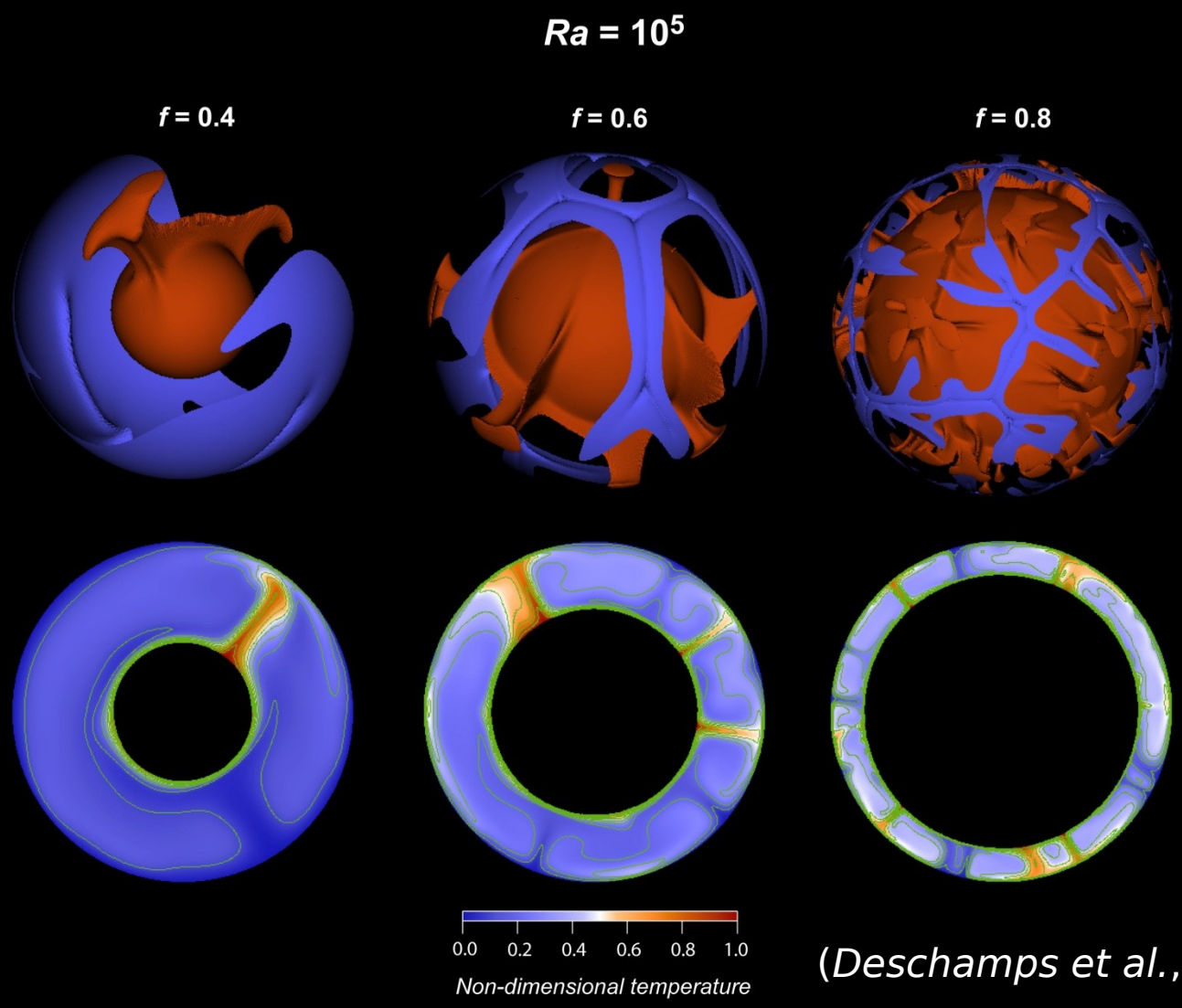
Convection rolls …



3D-Cartesian: influence of aspect ratio



Effect of the curvature



(*Deschamps et al.*, 2010a)

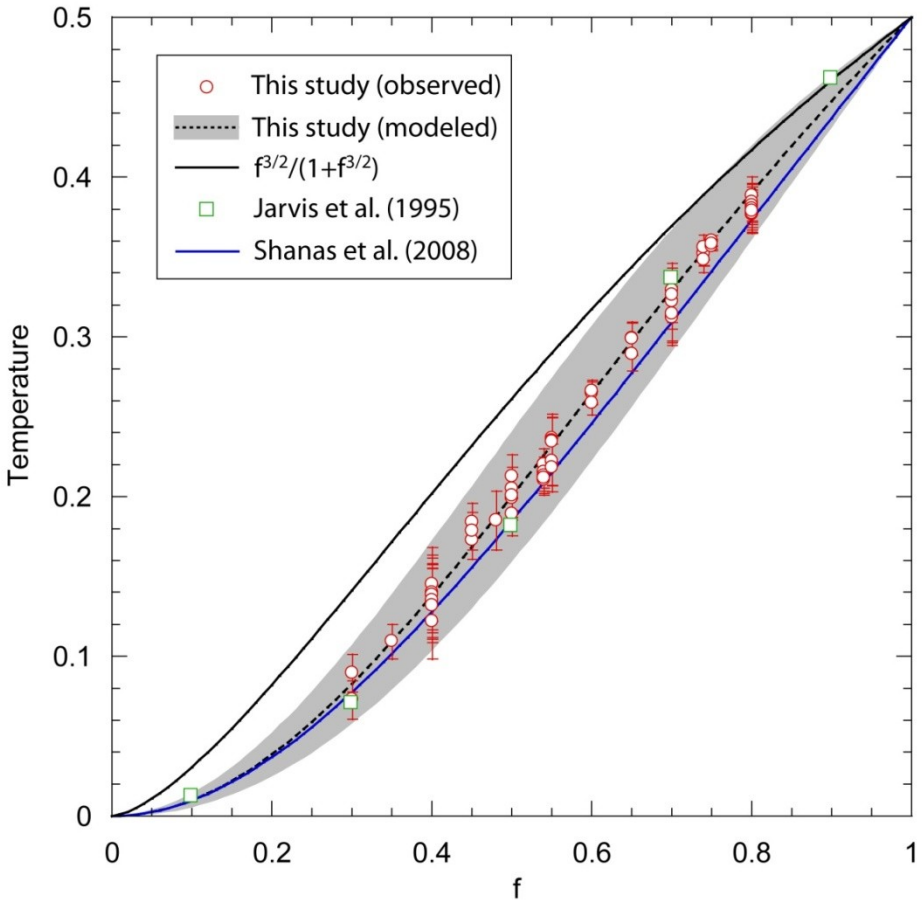
Effect of curvature on average temperature

- In Cartesian geometry, for bottom heated, isoviscous fluid, the volume average temperature is equal to 0.5.

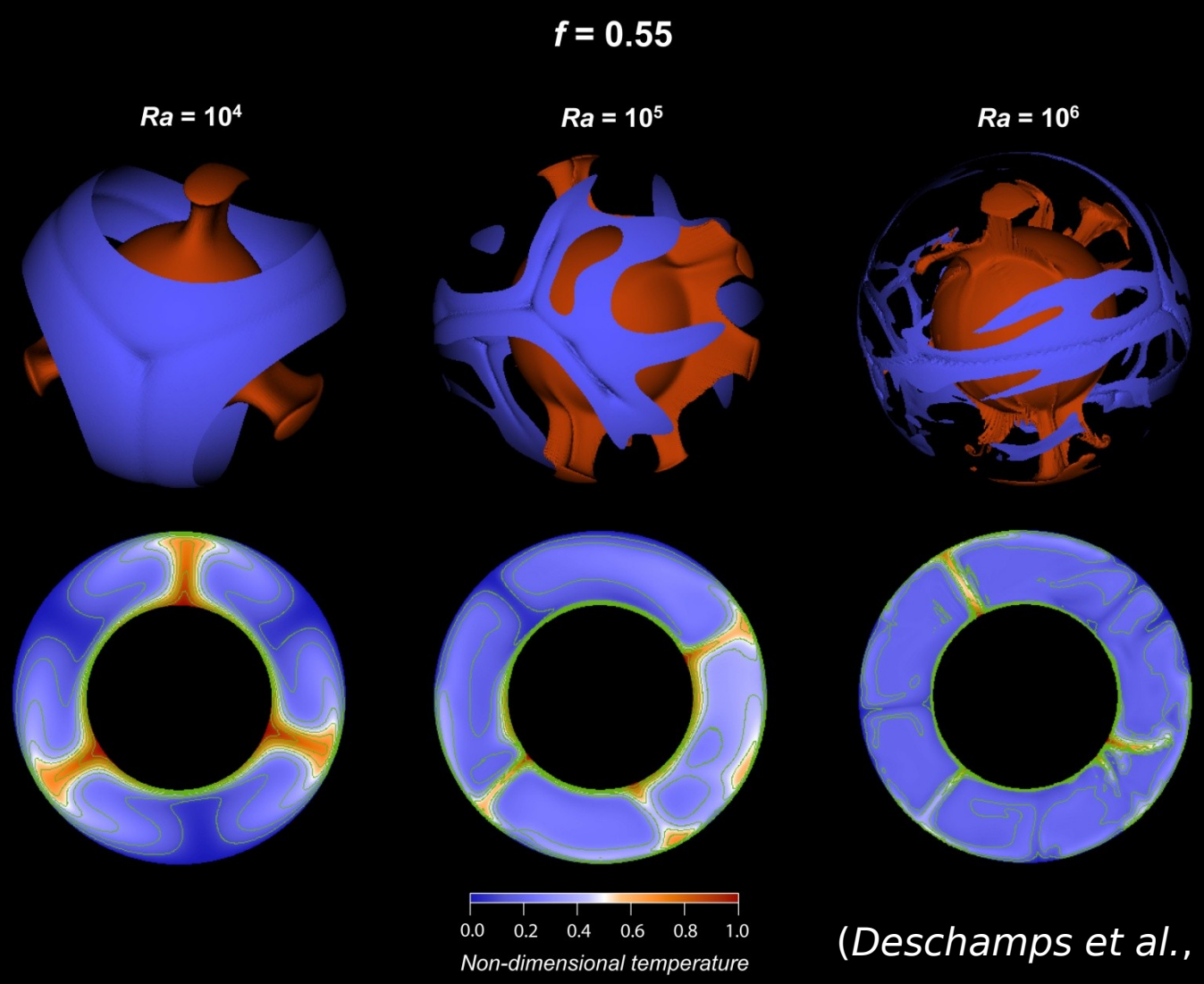
- In spherical geometry calculations (including from *Deschamps et al.*, 2010a) are well explained by:

f is the ratio between the inner and outer radii of the shell

- Extension of bottom TBL is reduced when *f* decreases, but temperature jump across it is larger.



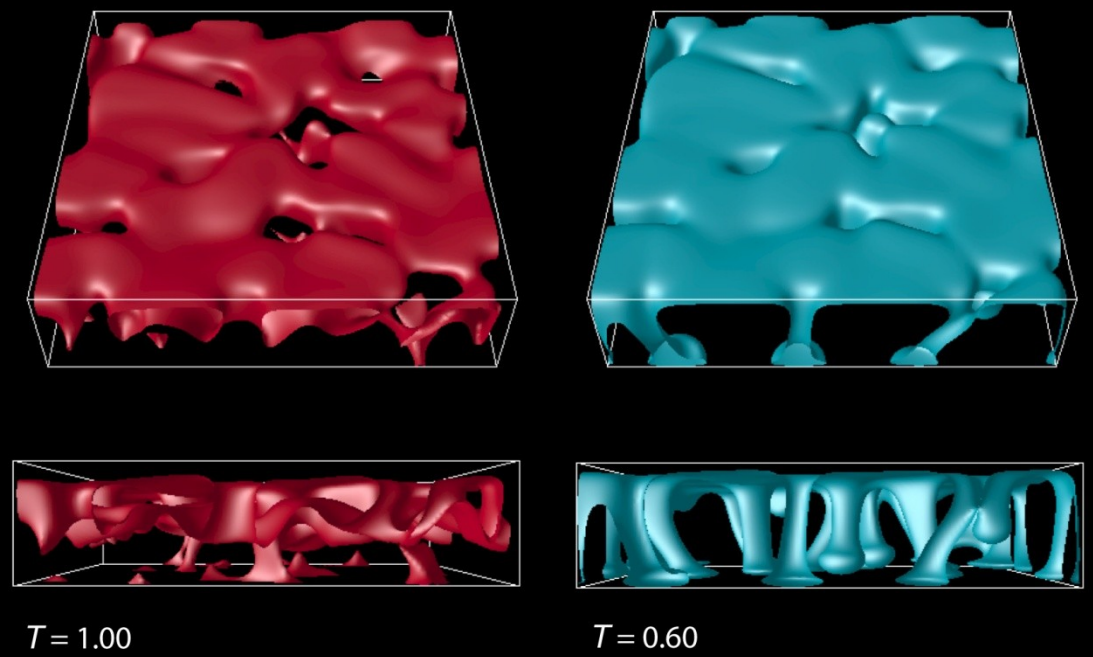
Effect of the Rayleigh number

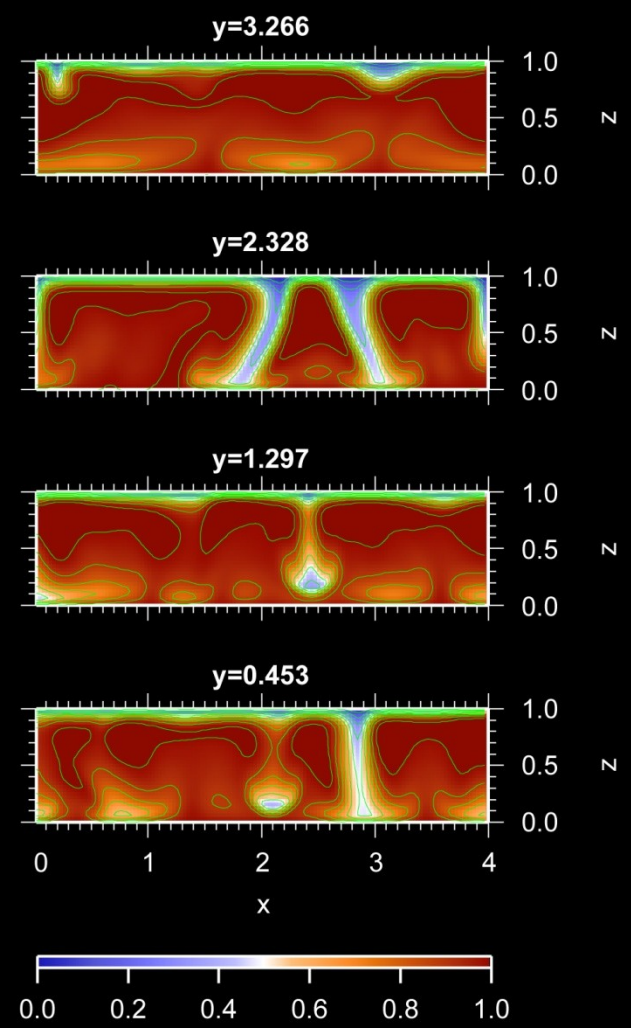


(*Deschamps et al.*, 2010a)

Influence of internal heating

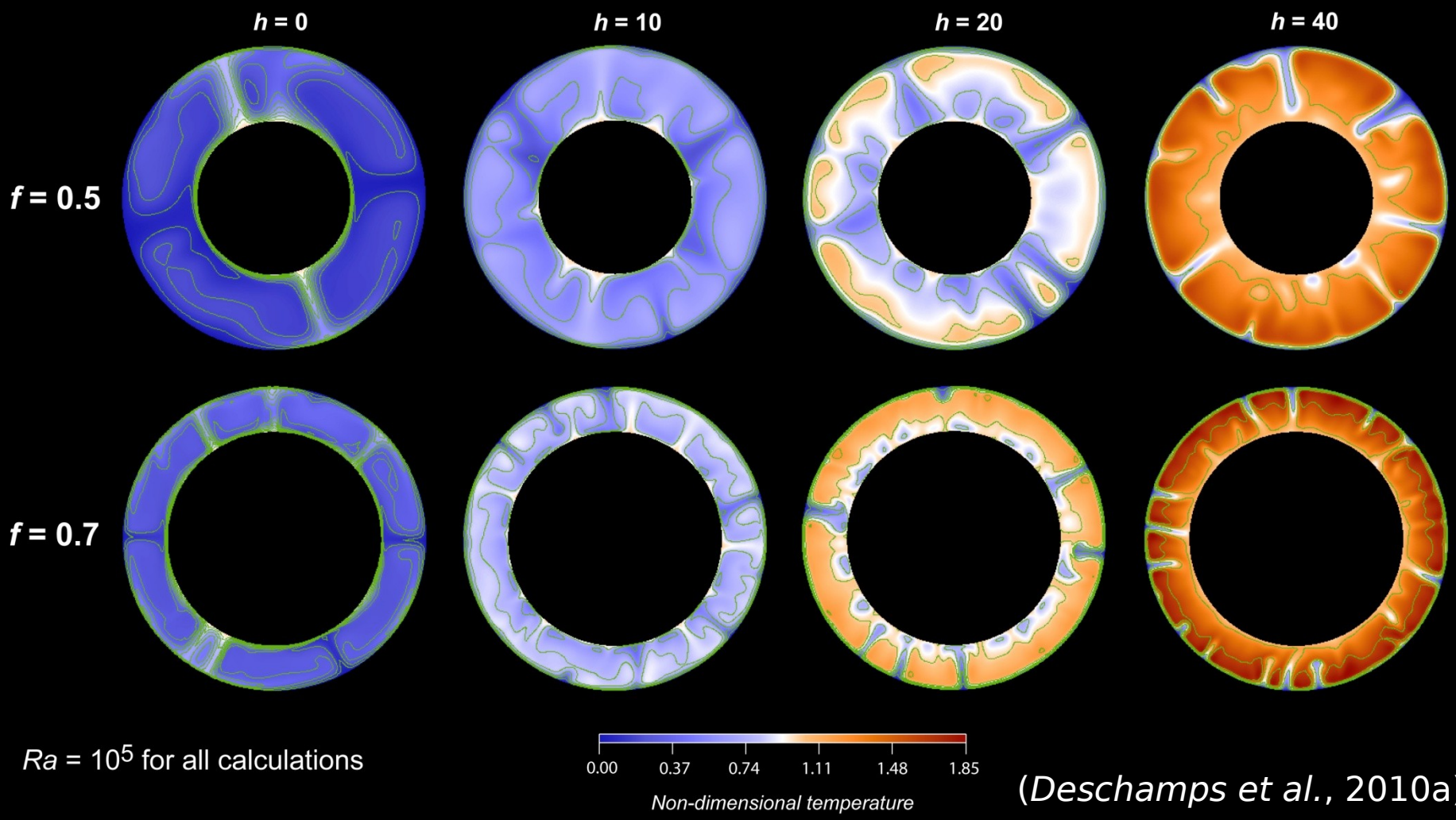
Ra = 10^5; isoviscous; H = 10 - Temperature





Plumes are less vigorous with increasing rate of internal heating. Heat transfer is progressively controlled by cold downwellings

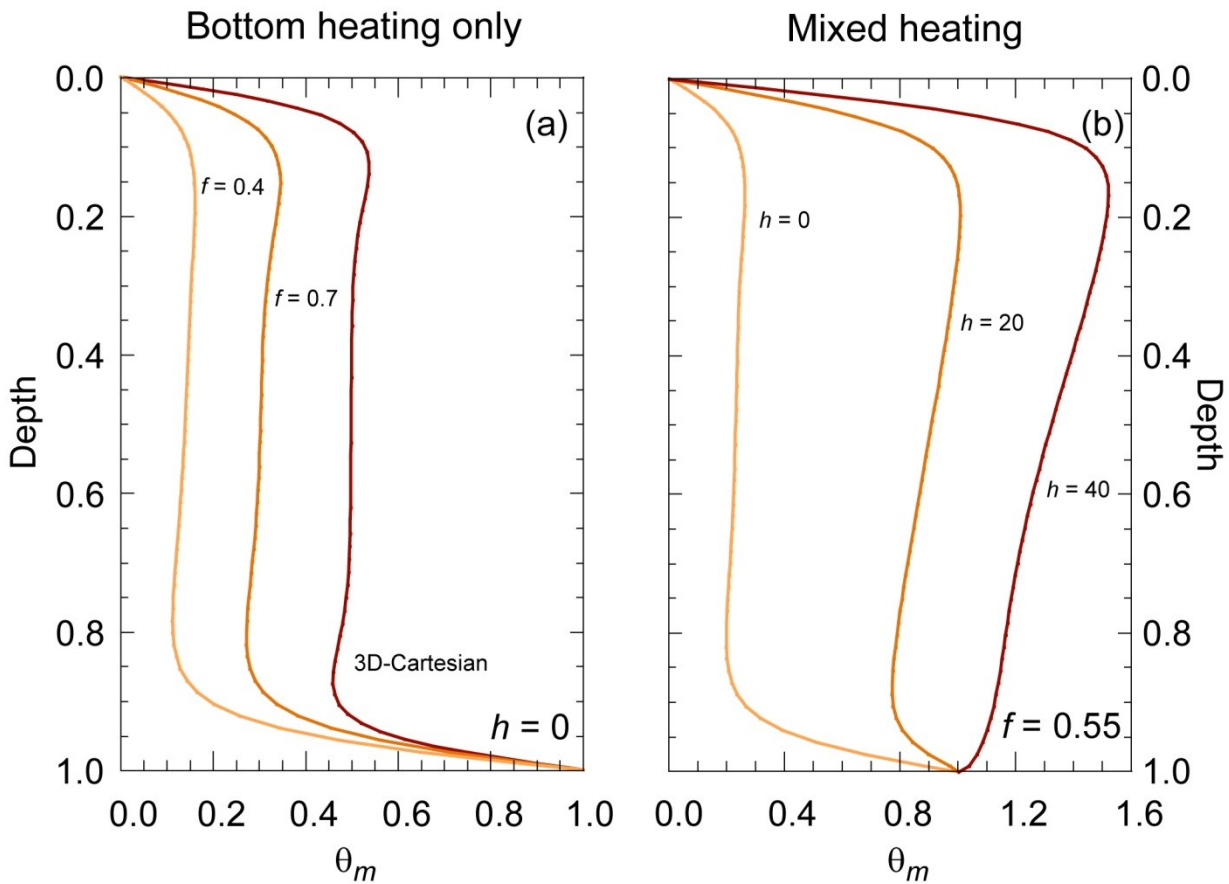
Influence of internal heating



$Ra = 10^5$ for all calculations

(*Deschamps et al.*, 2010a)

Mixed heating: temperature profiles



- The bottom thermal boundary layer progressively disappear with increasing amount of volumetric heating (h).
- Large values of *h* is too high: temperature profile is sub-adiabatic at the bottom.

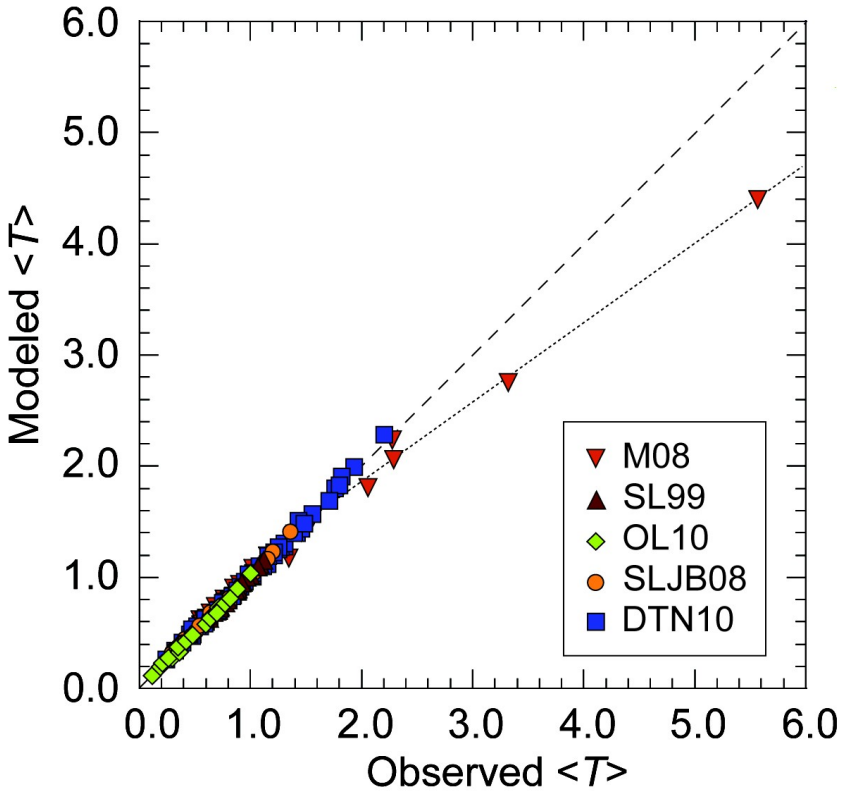
Mixed heating: average temperature

- Can be written as the sum of temperature for bottom heating, plus terms depending oh *h* and *Ra*:

Parameters fitting experiments in 2D-, 3D-Cartesian, and spherical geometries (*Deschamps et al*., submitted):

$a1 = 1.744 \quad b = 0.762$

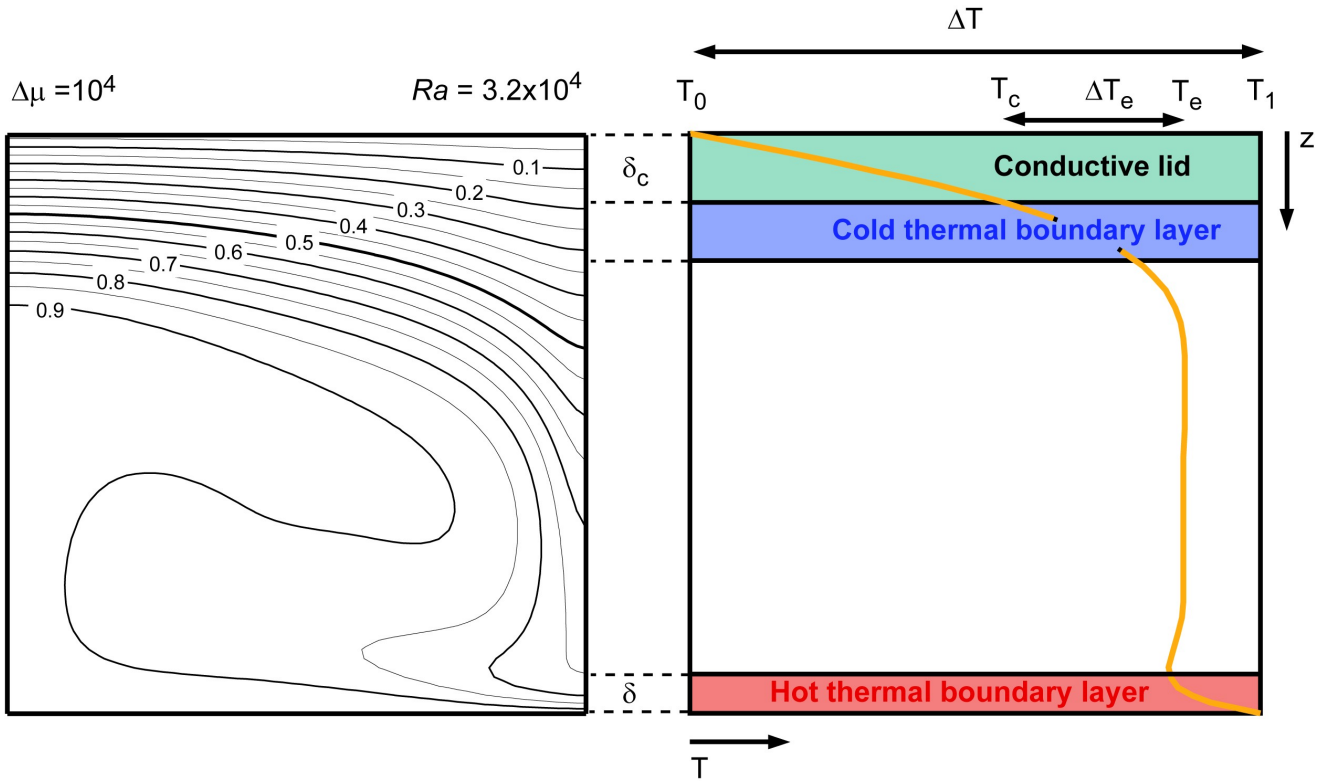
$a2 = -0.70 \quad c = 0.238$



- Valid only if *h* < *h*c, *h*c depending on Ra (see *Moore*, 2008 for 2D-Cartesian geometry; *Deschamps et al.*, submitted, for spherical geometry).

Stagnant lid regime

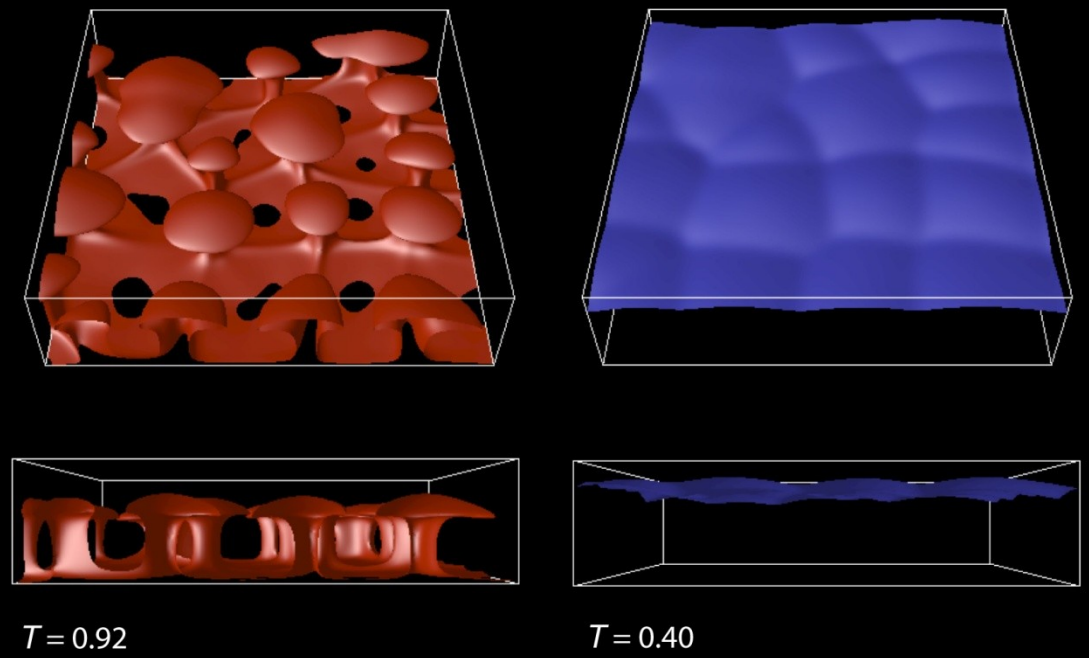
- For large (≥ 104) thermal viscosity contrast, a stagnant rigid lid forms at the top of the fluid.

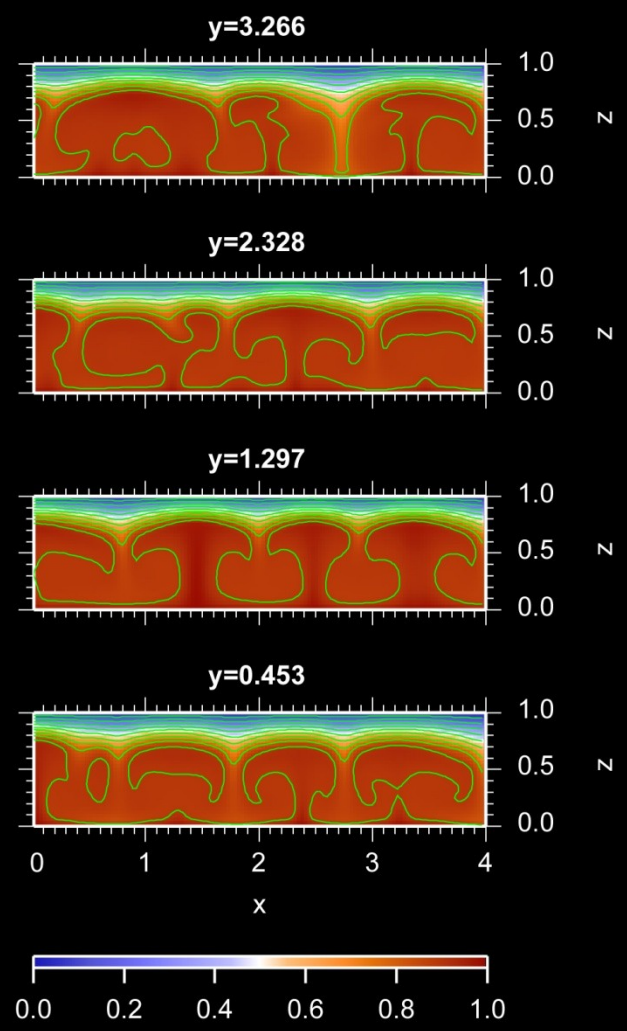


- In the lid, heat is transferred by conduction, which reduces the heat transfer across the whole fluid layer.

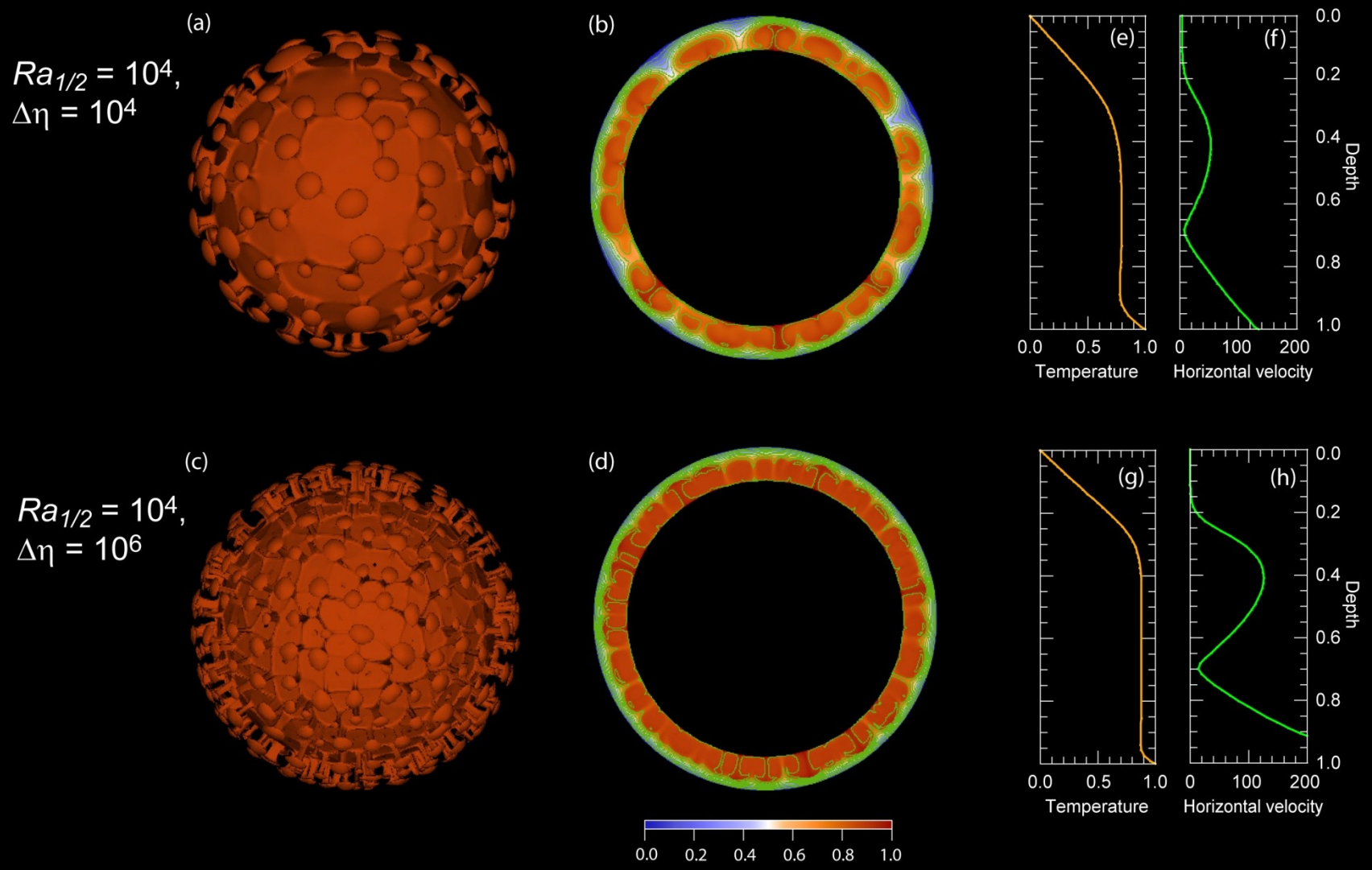
Effect of temperature dependent viscosity

$Ra_{1/2} = 10^4$; $\Delta\eta = 10^5$ - Temperature





Effect of temperature dependent viscosity



Stagnant lid regime: average temperature

- Viscous temperature scale:

Temperature jump in thermal boundary layer is proportional to a ΔTv (*Davaille and Jaupart*, 1993)

- Bulk non-dimensional temperature θm:

$$\left(1-\theta_m\right) = a\frac{\Delta T_v}{\Delta T}$$

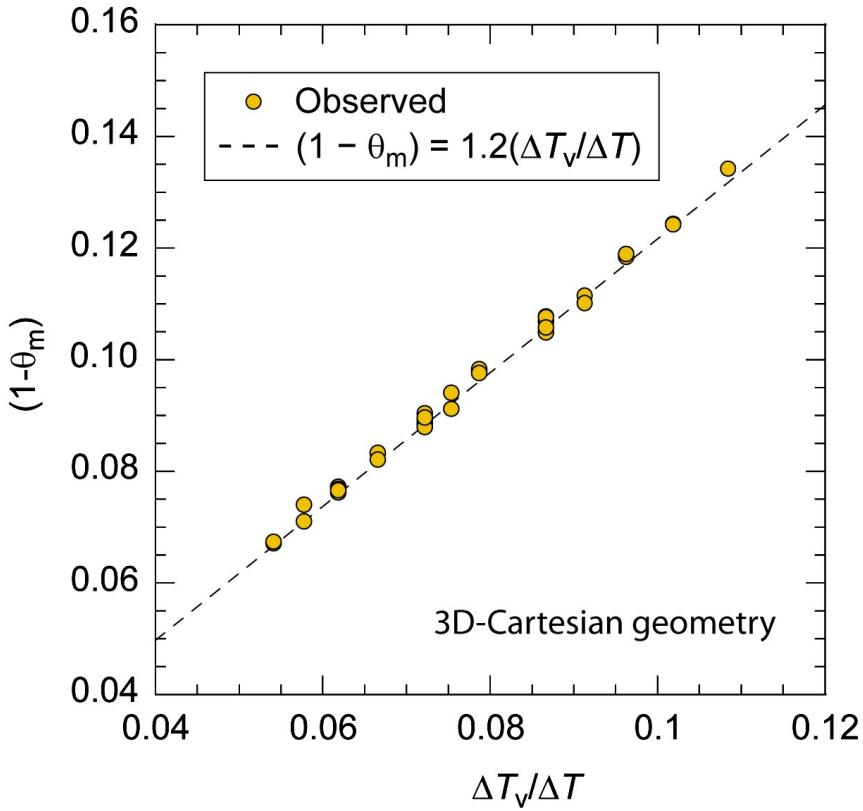
- 2D-Cartesian geometry (*Deschamps and Sotin, 2000)*:

  a = 1.43

- 3D-Cartesian geometry:

  a = 1.20

- Spherical geometry: depends on core siize; work in progress …



Bulk temperature increases with increasing viscosity contrast.

Stagnant lid regime: heat flux

- Heat flux (Nusselt number) *vs* Rayleigh number:

  *Ra*m: bulk Rayleigh number, calculated at ηm = η(*T*m)

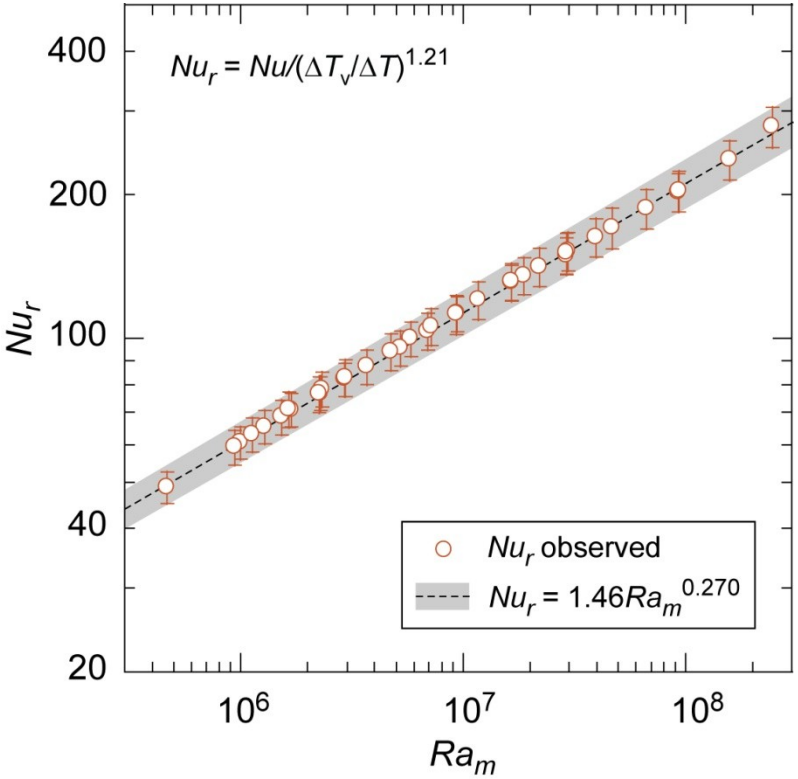
- 2D-Cartesian geometry (*Deschamps and Sotin, 2000)*:

  a = 3.8 *b* = 0.258 and *c* = 1.63

- 3D-Cartesian geometry:

  a = 1.46 *b* = 0.270 and *c* = 1.21

- Spherical geometry: work in progress …



Heat flux decreases with increasing viscosity contrast

Application to icy moons

The role of anti-freeze compounds in maintaining sub-surface ocean in icy moons interiors

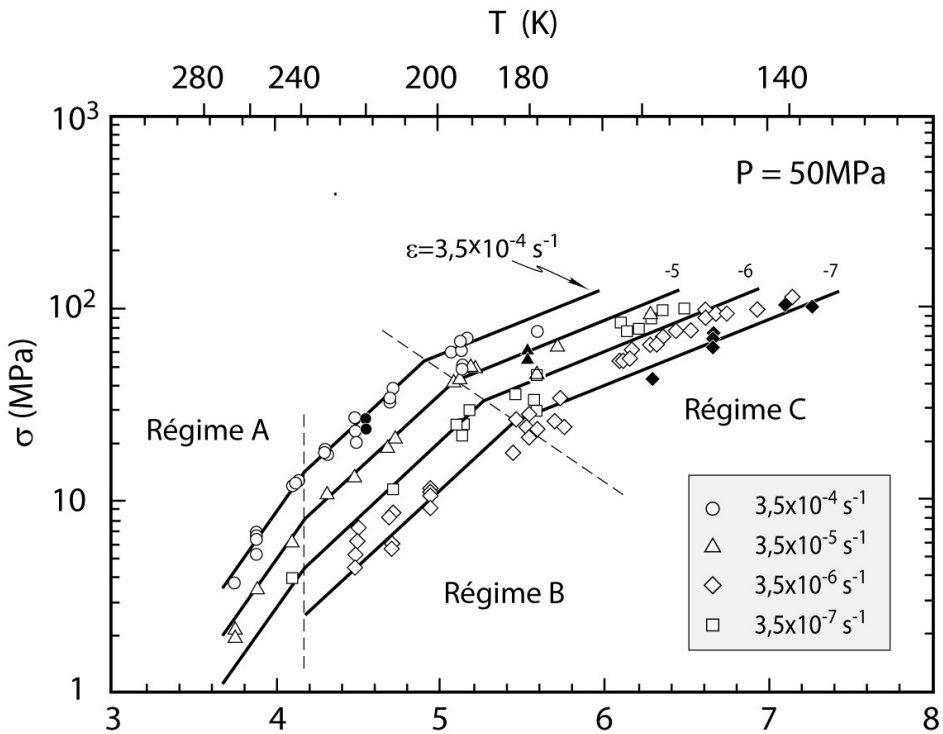
Viscosity of ices

- Ice viscosity is strongly temperature dependent:

$$\dot{\varepsilon}(T,P,\sigma) = A\sigma^n \exp\left[-\frac{E_a + PV_a}{RT}\right]$$

$$\eta = \sigma / 2\dot{\varepsilon}$$

- Three flow regimes, depending on temperature and strain rate (*Durham et al., 1997)*:



(*Durham et al.*, 1997)

- Non-newtonian rheology (n ≠ 1).

Parameterization for newtonian fluid (n = 1) may be used to model non-newtonian fluid (n = 1) using larger value of Ea (Dumoulin et al., 1999).

Convection in ice I shells may undergo stagnant lid regime

Internal heating

- Radiogenic heating in rocky cores (Callisto) and shells (Ganymede); Negligible in icy shells (e.g., outer ice I shells).

- Tidal heating is important for . . .

. . . bodies in non-synchronous rotation state (early stages of evolution)

. . . synchronously rotating satellites and

- Moving on high eccentricity orbit
- Close to parent planet
- High internal temperature

→ *Io (volcanism), Europa, Enceladus (geyser), Titan ?*

Tidal heating and radiogenic heating may be important to model the full evolution of moons.

Outline of calculations

Vigor of convection and heat transfer through an ice I shell heated from the bottom, and with strongly temperature dependent viscosity.

1. Thickness of outer ice I layer: fix the **bottom temperature** *(melting temperature of the mix water + impurities)*.

   - *Depends on the wt fraction of impurities in the primordial ocean.*
   - *Fraction of impurities re-calculated for each thickness.*

2. **Bulk temperature** from scaling law: $T_m = T_{bot} - 1.43\Delta T_v$

3. **Bulk viscosity**:

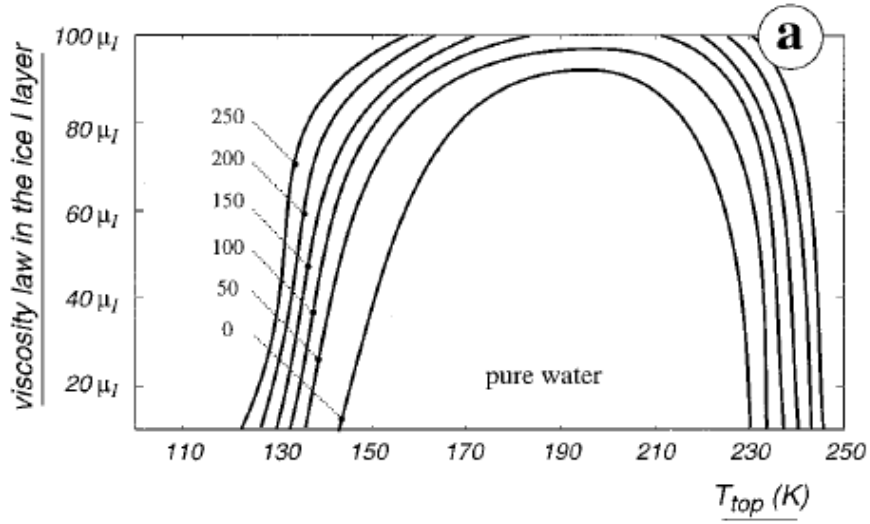
   with viscous temperature scale :

4. **Bulk Rayleigh number** of the ice I layer:

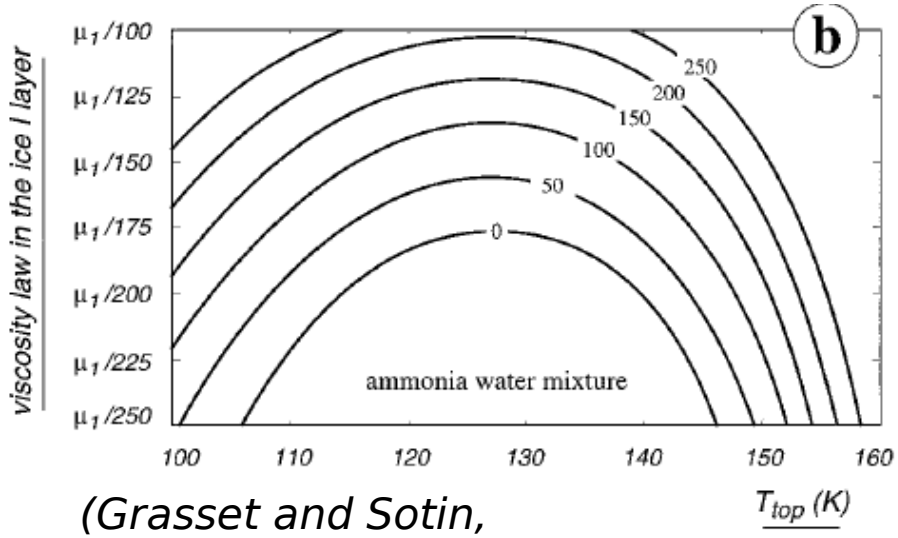
5. **Surface heat flux**: $\Phi_{surf} = 3.8\Phi_{cond} Ra_m^{0.258} \left(\frac{\Delta T_v}{\Delta T} \right)^{1.63}$ *with*

Thickness of liquid shell after 1Gyr evolution . . .

- Pure water: crystalization of the ocean can be completed within ~ 1Gyr



- Water + 15% NH3: crystalization can be completed only if the viscosity of ice I is smaller by 2 orders of magnitude.



(Grasset and Sotin, 1996)

The role of impurities

- Impurities (NH_3, CH_3OH, …) reduces the temperature of crystallization *(i.e., the temperature at the bottom of the ice I layer)*.

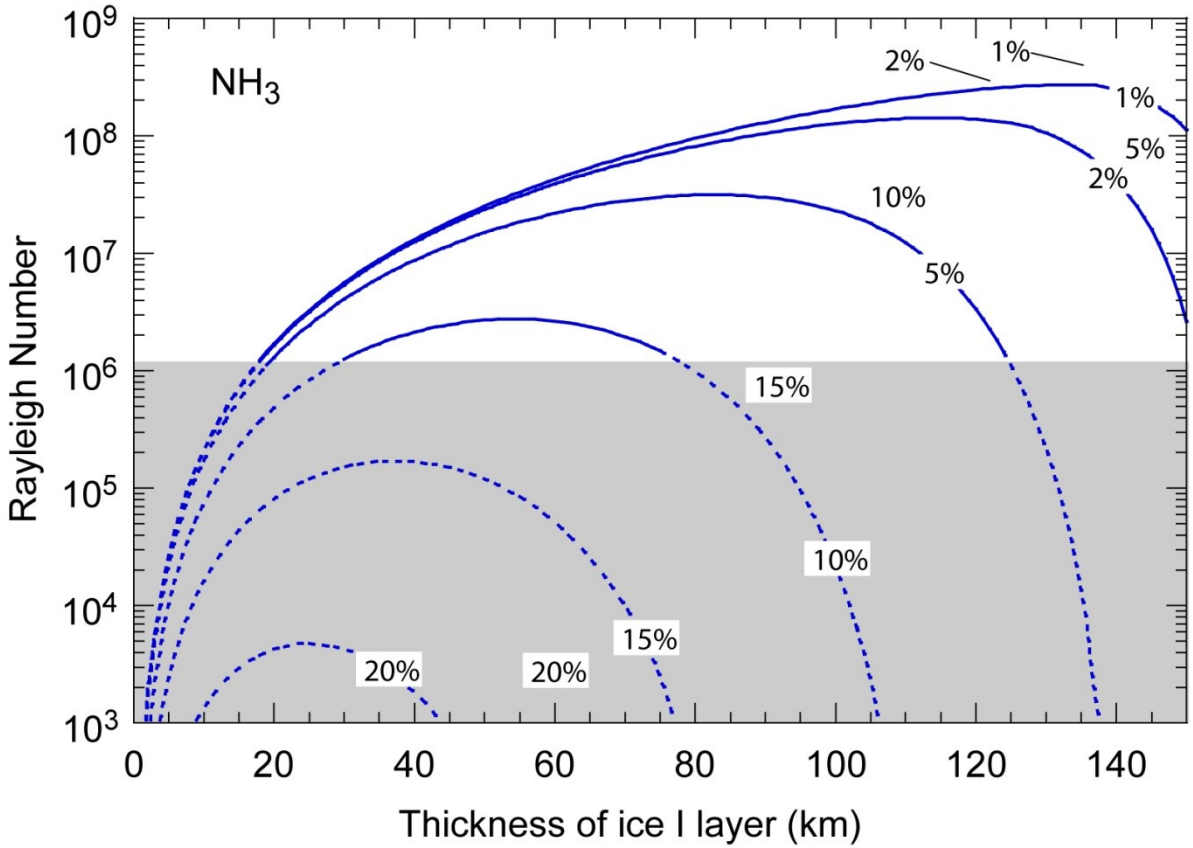
- This increases the bulk viscosity, and deceases the super-adiabatic temperature jump . . .

- . . . and the bulk Rayleigh number decreases. The vigor of convection and efficiency of heat transfer decrease. See calculations for ammonia *(e.g., Grasset and Sotin, 1996; Deschamps and Sotin, 2001)*

- Up to eutectic composition, pure water crystalizes, and impurities remain in the ocean, which further reduces bottom temperature at the bottom of ice layer

If the outer ice layer is too thick, convection stops. Crystallization of the primordial ocean may also stops, leaving a sub-surface ocean.

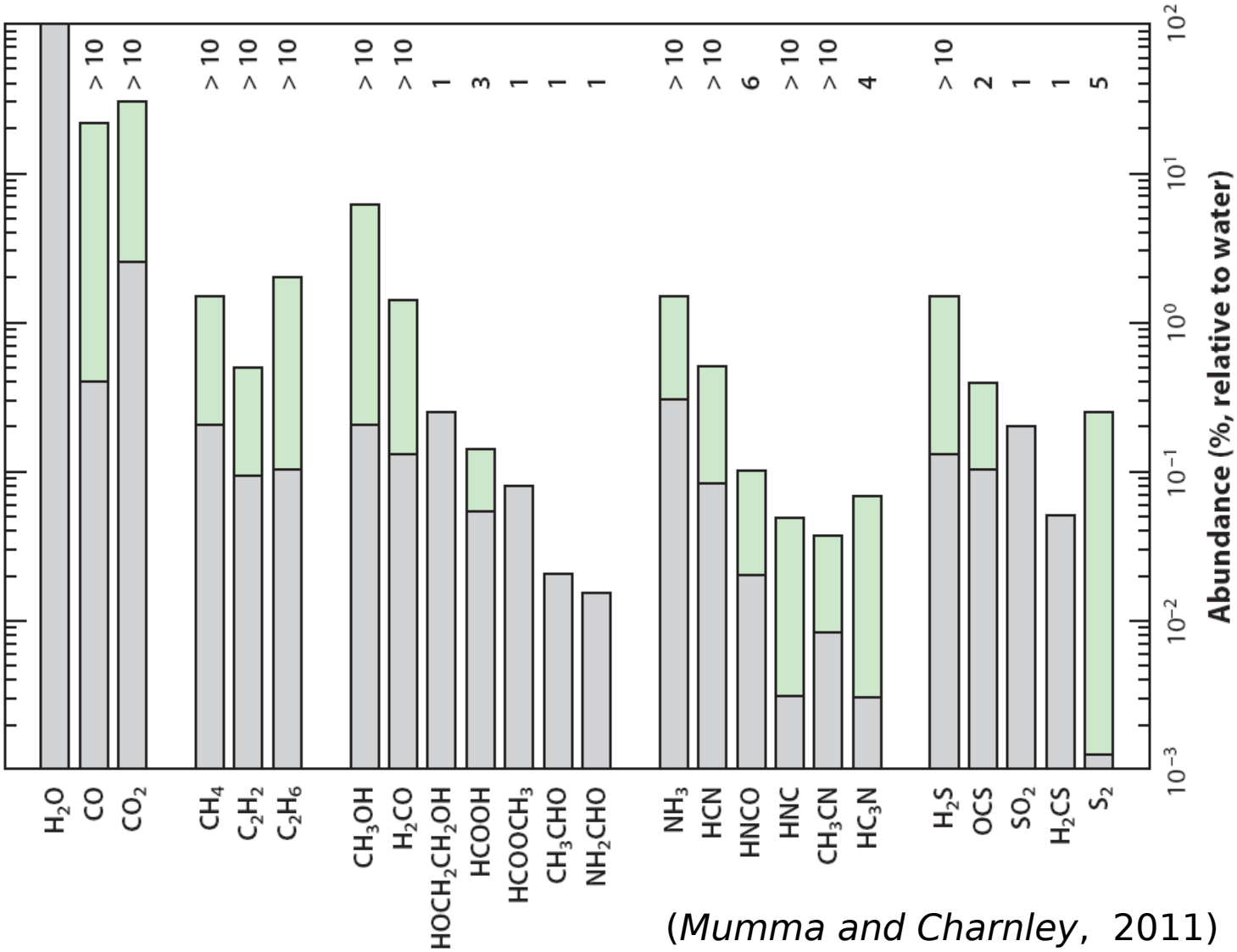
The influence of ammonia



(Deschamps and Sotin, 2001)

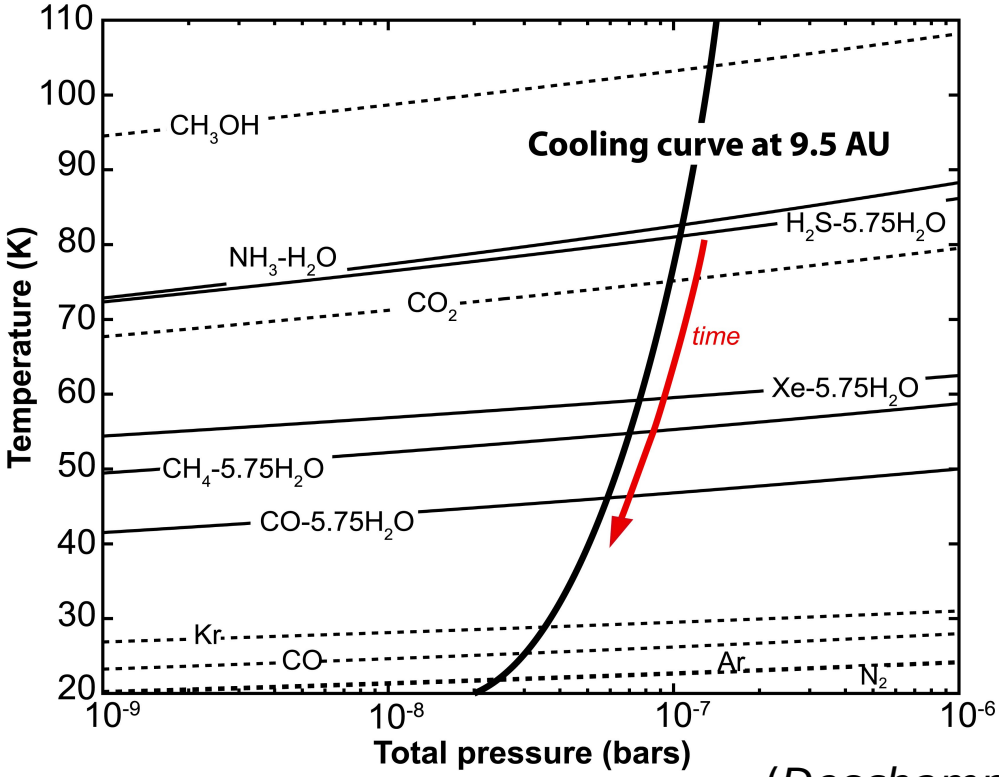
- For 5% NH3, at Titan's conditions: *h*crit = 125 km and *h*ocean ~ 125 km.

Volatiles in comets



(*Mumma and Charnley*, 2011)

- Condensation and chemical model in Solar Nebula at 9.5 AU from *(Mousis et al., 2009)*.



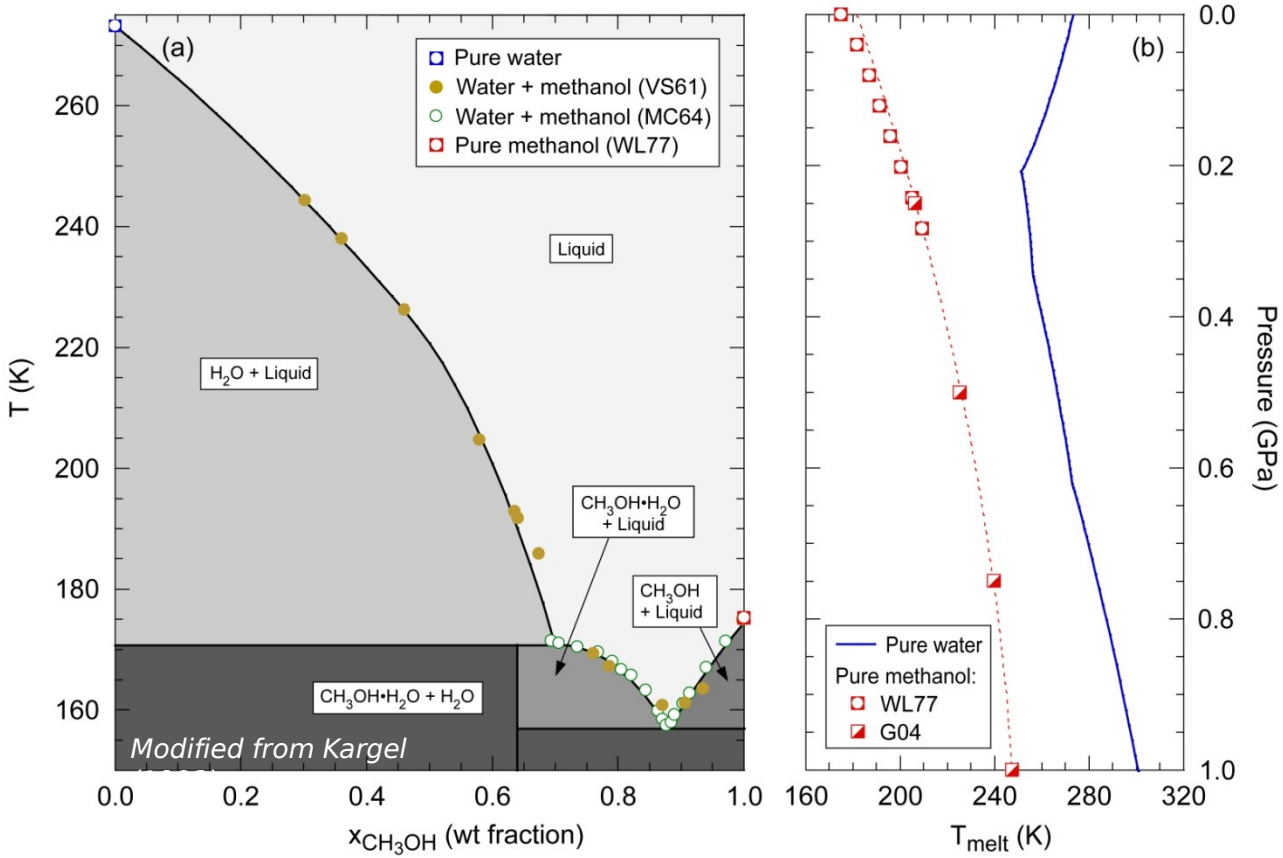
(*Deschamps et al.*, 2010b)

- *m*NH3 = 1 wt% and *m*CH3OH = 4 wt%

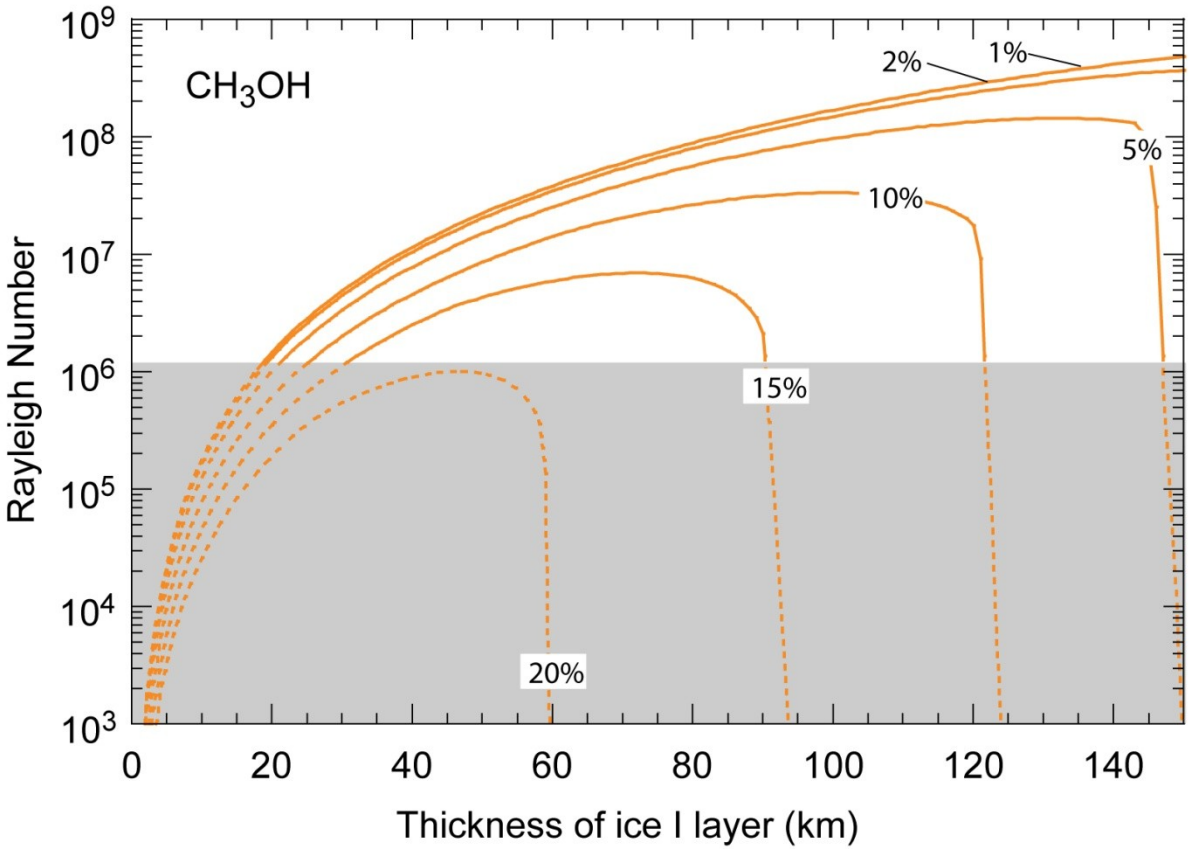
Consistent with abundances observed in comets *(e.g., Bockelée-Morvan et al., 2005; Mumma and Charnley, 2011)*.

Water-methanol system: phase diagram

- Well know at *P* = 0 *(Vuillard and Sanchez, 1961; Miller and Carpenter, 1964)*. Pure methanol freezes at 175 K, and eutectic mix (*x*CH3OH ~ 0.88) around 150 K.
- Poorly known at high pressure. Pure methanol freezes at 205 K and 248 K at 0.25 GPa and 1.0 GPa, respectively *(Würflinger and Landau, 1977; Gromnitskaya et al., 2004)*.

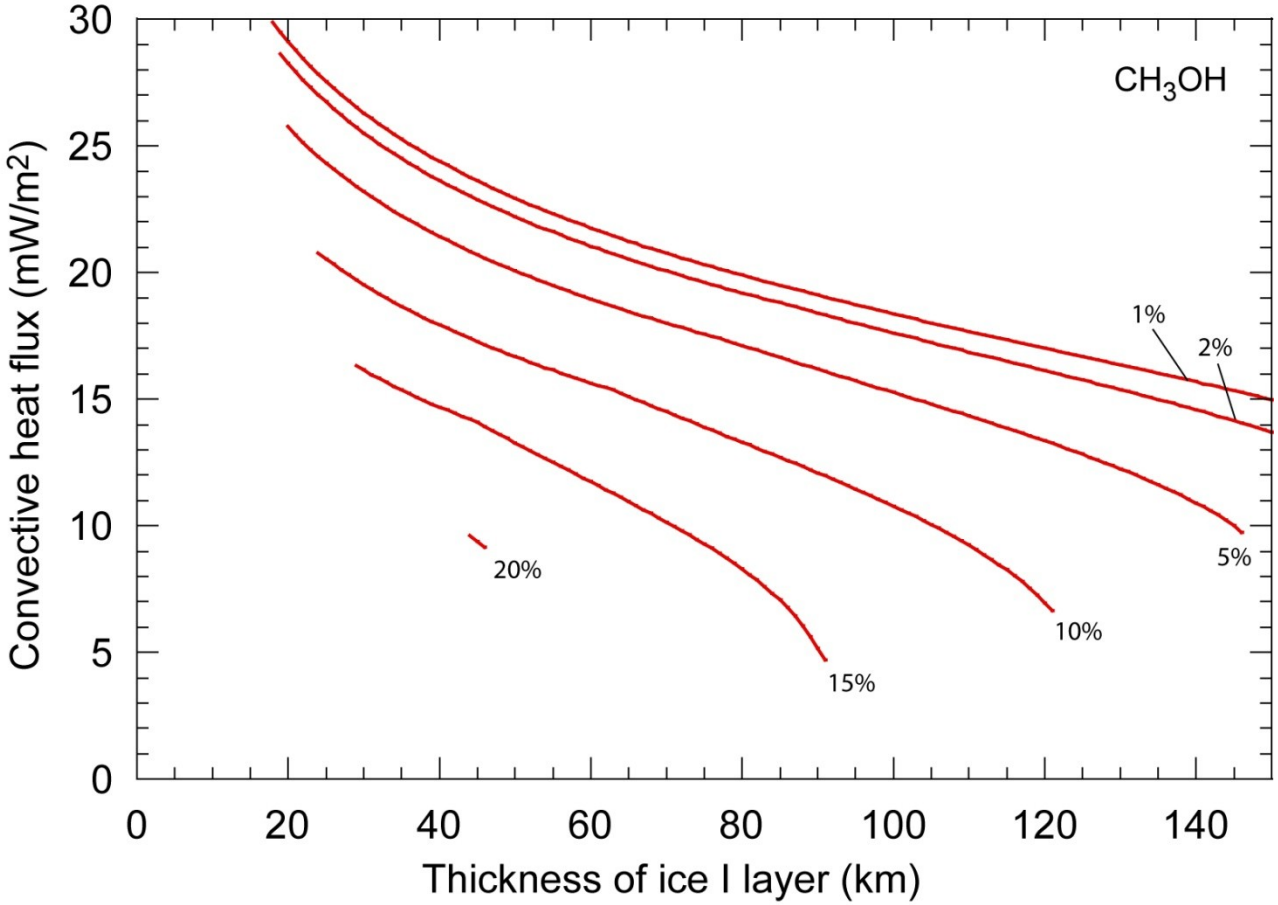


The role of methanol *(Rayleigh number)*



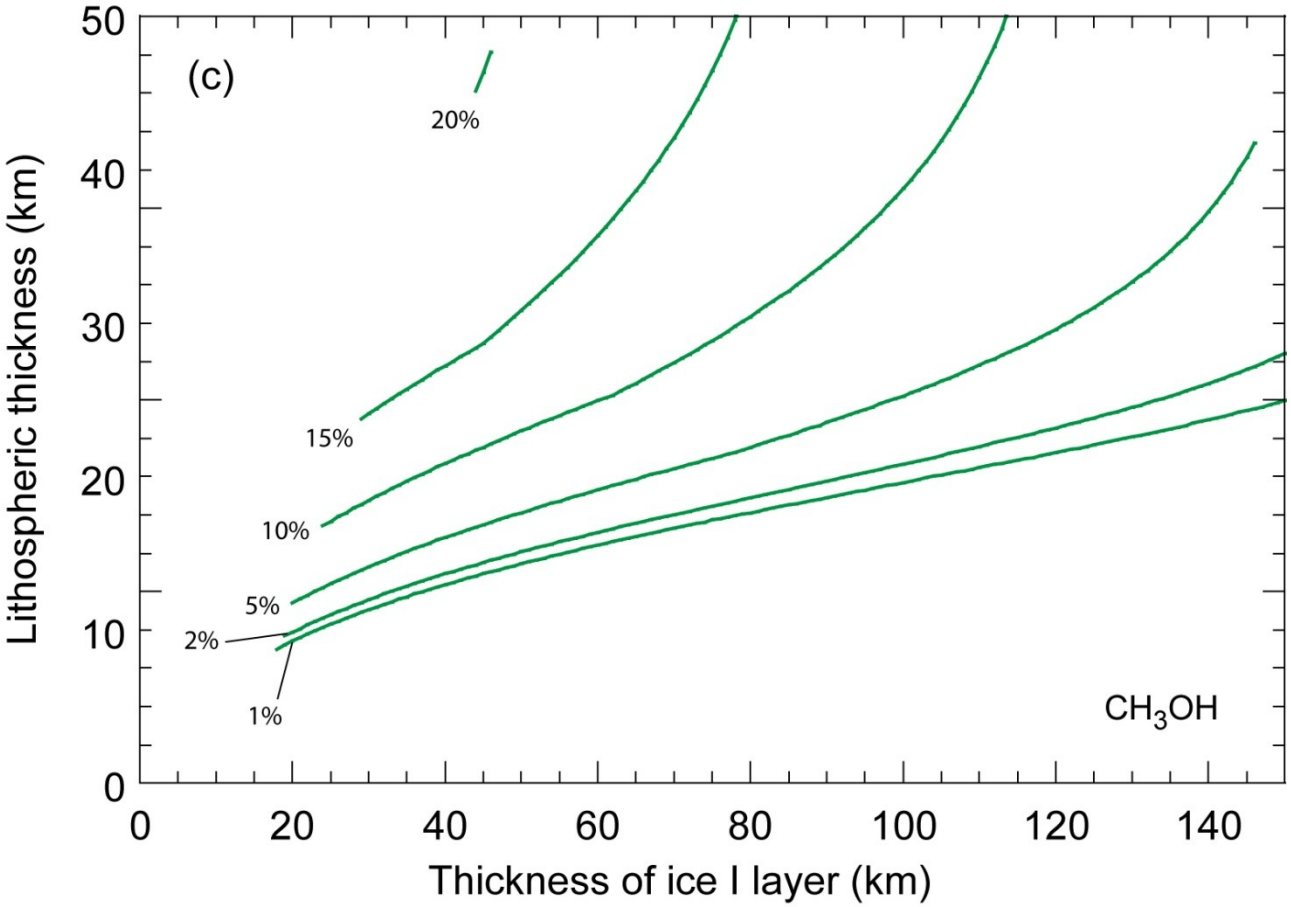
- The effect of 4 wt% methanol is similar to that of 2 wt% ammonia *(Deschamps et al., 2010)*.
- Application to Titan: 1 wt% ammonia and 4 wt% methanol may result in a 140 km thick ice I layer and 90 km thick ocean *(Deschamps et al., 2010b)*.

The role of methanol *(Heat flux)*



- 4 wt% methanol decreases the convective heat flux by ~ 10%. Less heat can be extracted from the ocean.

Lithospheric thickness



- Strongly increases with fraction of methanol. Strong, rigid lithosphere may explain lacks of tectonics e.g., at Callisto.

Titan: interior evolution and degassing

- Writes energy balance at each boundary between 2 layers.

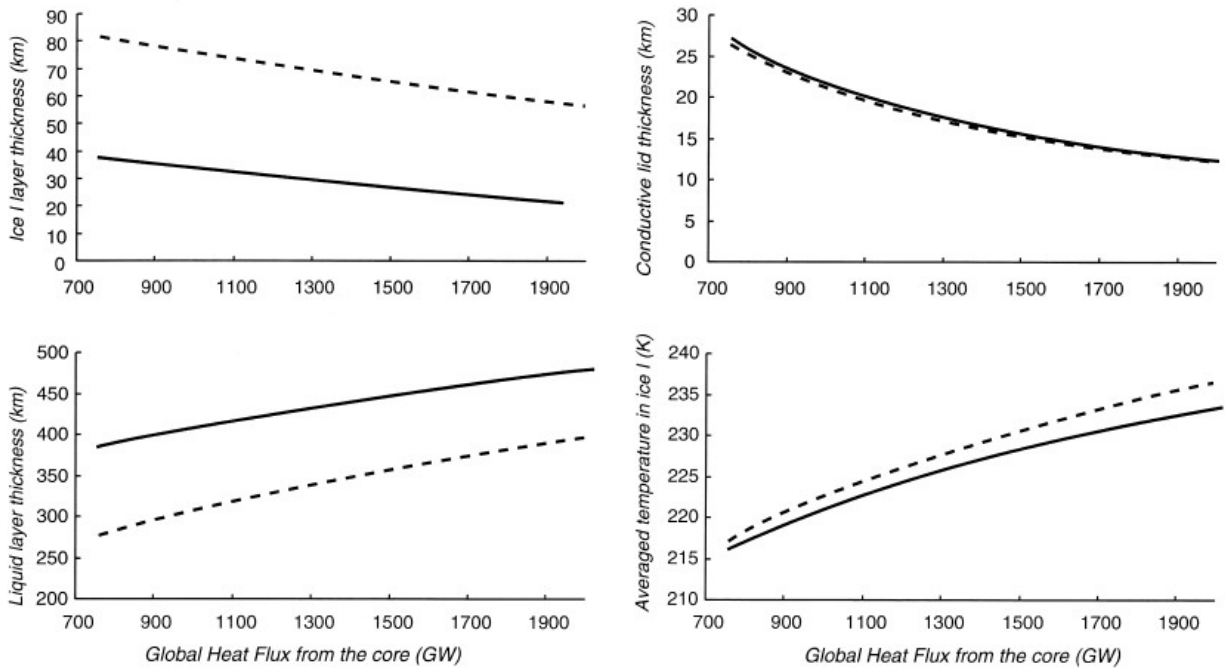


Fig. 9. Internal structure of the icy mantle of Titan as a function of the global heat flow expelled from the core for two distinct amounts of ammonia: dashed lines, 5%; plain lines, 15%. A global heat flux of 1.9 TW corresponds to the highest possible energy that can be expelled per unit time by the core once vigorous convection has started. At present, the heat flow is smaller and probably around 0.7 TW.

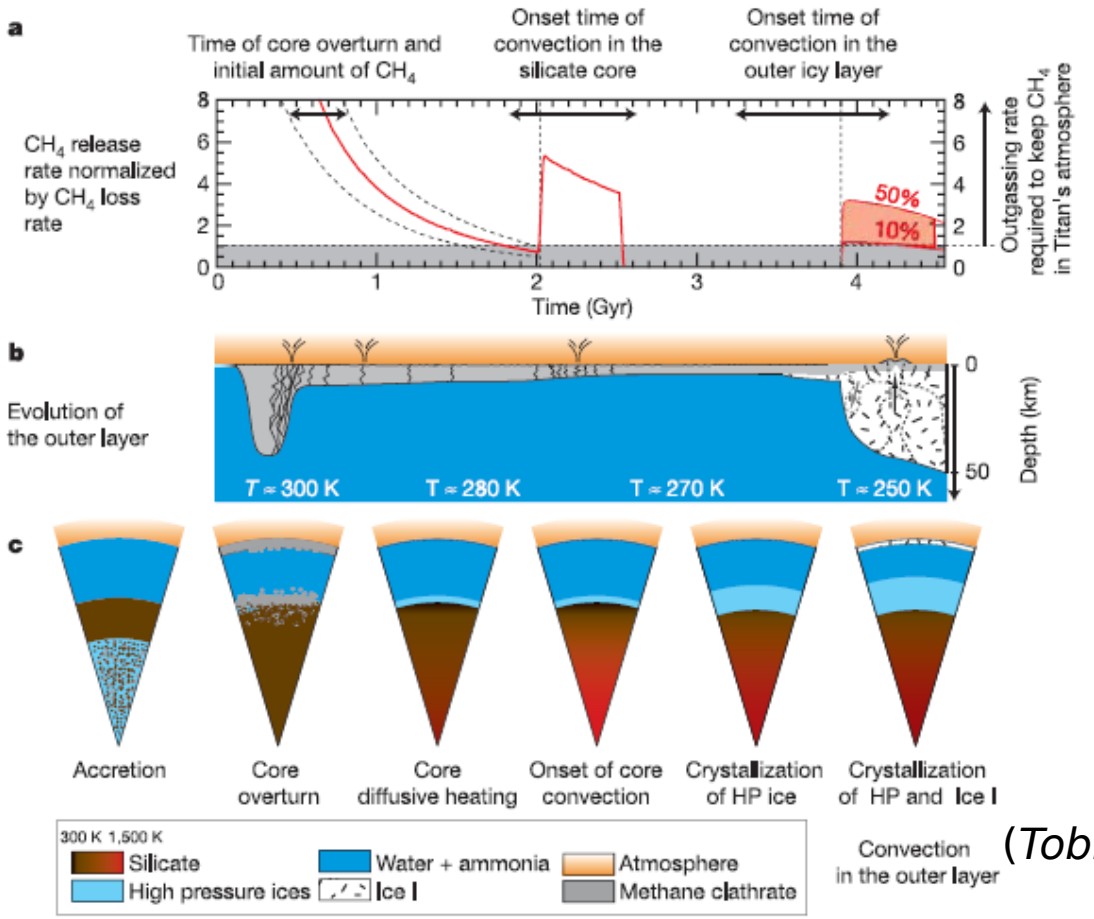
- Two scenarios, depending on the composition of the ocean:

- Pure H2O: small decrease of temperature at the bottom of ice I, vigorous convection complete crystalization

- H2O + NH3: strong decrease of temperature at the bottom of ice I, weak convection, crystalization is slow and stops.

Titan: interior evolution and degassing

- Methane is stored as clathrates beneath the crust.
- Several outgassing episodes occur, the last one related to crystallization and convection of ice I layer



(*Tobie et et al.*, 2008)

Conclusions and perspectives

Dynamics of outer ice shell of icy moons

- Primordial oceans crystallizes at their top and at their bottom.

- The presence of impurities (*e.g.*, NH3, CH3OH) decreases the vigor of convection and the heat transfer.

- This may help maintaining ocean between ice I and high pressure ice layers.

- Preliminary estimate for Titan: 1 wt% NH3 and 4 wt% CH3OH may maintain an ocean at least 90 km-thick beneath an ice I layer at most 140 km-thick.

Perspectives:

- Ice properties: phase diagram of the system water-methanol, thermo-elastic properties.

- Modeling of thermal convection: stagnant lid convection in spherical geometry.

- Application to icy moons: evolution of satellites' interiors

- Application to other bodies: KBO

Mass conservation *(continuity)*

- Lagrangian formulation:

$$\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + \frac{\partial x_j}{\partial t}\frac{\partial \rho}{\partial x_j} = \frac{\partial \rho}{\partial t} + \mathbf{u}\nabla\rho = -\rho\nabla\cdot\mathbf{u}$$

ρ, density *u, velocity*

- Incompressible fluid:

$$\frac{D\rho}{Dt} = 0 \quad \Rightarrow \quad \nabla\cdot\mathbf{u} = 0$$

- Anelastic, compressible fluid:

$$\frac{D\rho}{Dt} \neq 0 \quad \text{but} \quad \frac{\partial \rho}{\partial t} = 0$$

$$\Rightarrow \quad \nabla\cdot(\rho\mathbf{u}) = 0$$

Used to model thermo chemical convection (e.g., Earth's mantle). Implies thermal expansion to vary with depth.

Differentiation

- After formation, heavier elements (here silicates + metals) migrate towards the center.

- Depending on accretion history (*e.g.*, amount of available heat), differentiation may be fully (Ganymede) or partially (Callisto) completed.

- Radial structure after differentiation: primordial ocean water + volatiles covering

- Metalic core + rocky mantle (Ganymede)
- Rock + metal core
- Mixed (rock + metal + ices) core (Callisto)

Strain and relaxation time

- Elastic strain: $$\tau_{ij} = \mu \varepsilon_{ij}$$

  μ, shear modulus (rigidity)
  τij, deviatoric stress tensor
  εij, strain tensor

- Viscous strain: $$\tau_{ij} = \eta \frac{\partial \varepsilon_{ij}}{\partial t}$$

  η, viscosity.
  τij, deviatoric stress tensor
  ∂εij/∂t strain rate tensor

- Material of planetary mantles (and icy shells) are visco-elastic . . .

  Visco-elastic strain:

$$\frac{\partial \varepsilon_{ij}}{\partial t} = \frac{1}{2\eta}\tau_{ij} + \frac{1}{\mu}\frac{\partial \tau_{ij}}{\partial t}$$

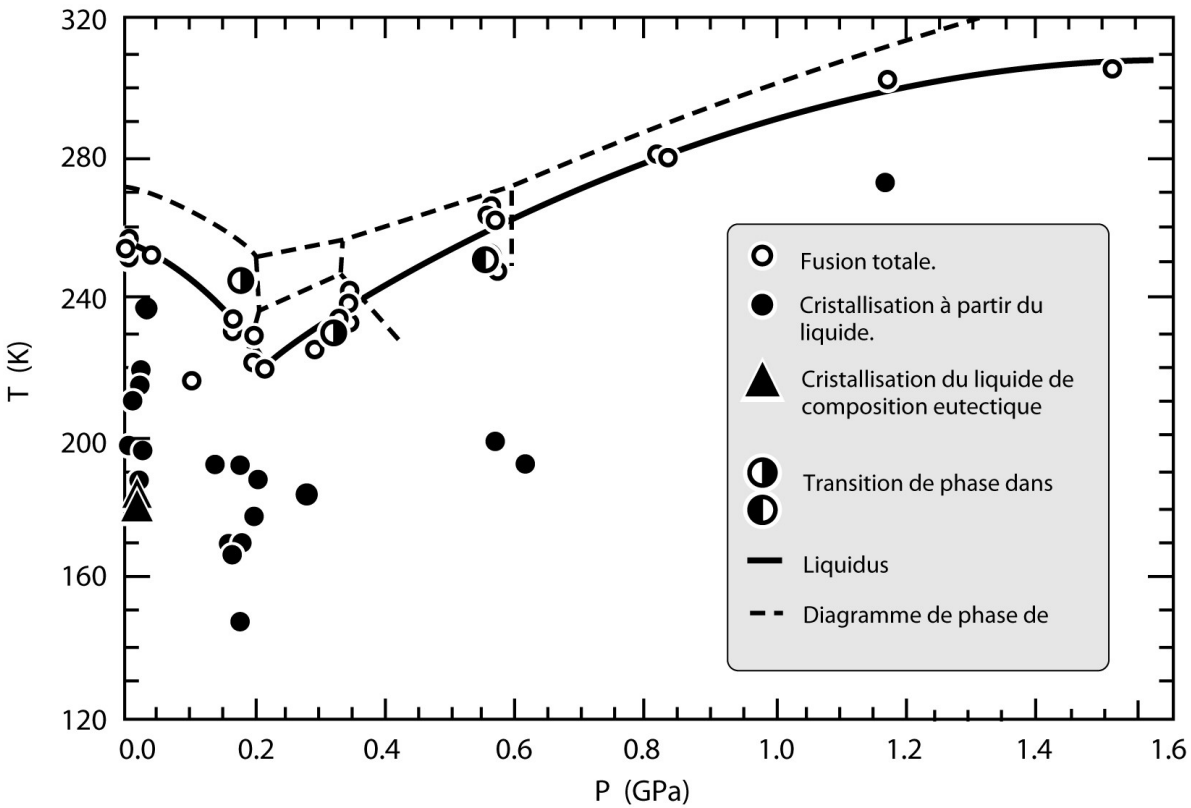
$$= \frac{1}{2\eta}\left(\tau_{ij} + 2t_r \frac{\partial \tau_{ij}}{\partial t}\right)$$

- tr = η/μ, characteristic relaxation time, ~ 2000 years for the Earth's mantle.
- tobs << tr : elastic deformation dominates (e.g. seismic waves)
tobs >> tr : viscous deformation dominates (e.g., mantle flow).

- Lithosphere: plastic and brittle deformation

Water-ammonia system: phase diagram

- Well know at P = 0 and higher pressures *(Grasset et al., 1995; Hogenboom et al., 1995)*. Eutectic mix (xNH3 ~ 0.32) freezes around 175-180 K, depending on pressure.
- 15% NH3 + 85% H2O *(Grasset et al., 1995)*.



Volatiles abundances from chemical model

- Titan's building blocks formed in Solar Nebula (instead of Saturn's sub-nebula) as indicated by D/H ratio at Enceladus *(Mousis et al., 2009)*.

- Chemical model:

- Cooling curve at 9.5 AU from accretion disk model of *Papaloizou and Terquem (1999)*.

- Stability curves: hydrates and clathrates from *Lunine and Stevenson (1985)* compilation; pure condensates from *Lide (2002)*.

- Equilibrium curve for CH3OH ice (not known for CH3OH clathrates).

- CH3OH stable below ~ 100 K at 10-7 bars.

- Mass fraction of volatile *i* determined from the ratio:

Elements abundances from *Lodders (2003)*.

X = mass mixing ratio;
Σ = surface density in sub-nebula