Cosmological implications of inhomogeneities in intra-cluster gas

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- Professor: Yasushi Suto, Assistant Professor: Atsushi Taruya + 8 graduate students (one female, one from mainland China)

- Cosmology
  - Dark energy and baryon acoustic oscillation
  - Galactic dust extinction map and FIR emission of galaxies
  - Soft X-ray emission from warm–hot intergalactic medium
  - Modeling density profiles of galaxy clusters

- Exoplanet
  - Spin-orbit misalignment using the Rossiter-McLaughlin effect
  - Modeling scattered light of a second earth
  - Dynamical evolution of a few body planet system
Collaborators and references

- E. Reese et al. (2010)
  - *Impact of Chandra Calibration Uncertainties on Galaxy Cluster Temperatures; Application to the Hubble Constant*

- Kawahara et al. (2007)
  - *Radial Profile and Lognormal Fluctuations of the Intracluster Medium as the Origin of Systematic Bias in Spectroscopic Temperature*

- Kawahara et al. (2008a)
  - *Systematic Errors in the Hubble Constant Measurement from the Sunyaev-Zel'dovich effect*

- Kawahara et al. (2008b)
  - *Extracting Galaxy Cluster Gas Inhomogeneity from X-ray Surface Brightness: A Statistical Approach and Application to Abell 3667*
What are galaxy clusters?

- **Abell (optical) clusters**
  - the Abell radius
  - \( m_3 < m < m_3 + 2 \)
  - richness class

- **SZ clusters**
  - \( \Delta I_{SZ} \propto n_e T_e \)

- **Press-Schechter halos**
  - spherical collapse
  - \( \Delta_{vir} = 18\pi^2 \)

- **X-ray clusters**
  - \( S_x \propto n_e^2 T_e^{1/2} \)

- **Halos in N-body simulations**
  - friend-of-friend
  - linking length = 0.2

Definitely they are closely related, but the exact one-to-one correspondence is unrealistic.
Fundamental limitation in cosmology with galaxy clusters

- **Cosmological parameters from...**
  - Observed cluster abundance as a function of $T$ and $z$
  - Predicted halo abundance as a function of $M$ and $z$

- $M_{\text{halo}} - T_{\text{cluster}}$ relation
  - $M_{\text{halo}}$: size of cluster? Non-sphericity?
  - $T_{\text{cluster}}$: non-isothermal, inhomogeneity in intra-cluster medium?
Mass-weighted, emission-weighted, and spectroscopic temperatures of clusters

\[
\langle T \rangle_w = \frac{\int T W dV}{\int W dV}
\]

Except for idealized isothermal case, cluster temperature is ill-defined

<table>
<thead>
<tr>
<th>definition</th>
<th>W (weight)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_m ) mass-weighted</td>
<td>( n )</td>
</tr>
<tr>
<td>( T_{ew} ) emission-weighted</td>
<td>( n^2 \Lambda(T) ) simulation</td>
</tr>
<tr>
<td>( T_{spec} ) spectroscopic</td>
<td>spectral fit observation</td>
</tr>
<tr>
<td>( T_{sl} ) spectroscopic-like</td>
<td>( n^2 T^{-0.75} ) Mazzotta et al. (2004)</td>
</tr>
</tbody>
</table>
Simulated clusters in the local universe

- **SPH simulations** by Dolag et al. (2005)
- **Local universe distribution** in a sphere of $r=110\text{Mpc}$
- Initial condition: smoothing the observed galaxy density field of IRAS 1.2 Jy survey (over $5h^{-1}\text{Mpc}$), linearly evolving back to $z=50$
- with cooling, star formation, SN feedback, and metallicity evolution in $\Lambda$CDM
Projected views of \textit{simulated clusters}

- Coma
- Perseus
- Virgo
- Centaurus
- A3627
- Hydra
$T_{\text{spec}}$ is systematically smaller than $T_{\text{ew}}$

- Spectroscopically more weight (more lines) toward cooler regions

- Mazzotta et al. (2004) & Rasia et al. (2005) found $T_{\text{spec}} \sim 0.7 \ T_{\text{ew}}$ from simulations

- We confirm their results using simulated clusters of Dolag et al. (2005) $T_{\text{spec}} \sim 0.8 \ T_{\text{ew}}$

(see also Mathiesen & Evrard 2001)

Origin of $T_{\text{spec}} < T_{\text{ew}}$ (1) mean radial profile

- Density and temperature radial profiles of simulated clusters
- Polytropic $\beta$ model

\begin{align*}
< n > (r) &= n_0 \left[ \frac{1}{1 + (r / r_c)^2} \right]^{3\beta/2} \\
< T > (r) &= T_0 \left[ < n > (r) / n_0 \right]^{\gamma-1}
\end{align*}

Origin of $T_{\text{spec}} < T_{\text{ew}}$ (2) Local inhomogeneity

- Local inhomogeneities of density and temperature of simulated clusters
  - $\delta_n = n(r, \theta, \phi) / \langle n \rangle(r)$
  - $\delta_T = T(r, \theta, \phi) / \langle T \rangle(r)$
- Log-normal PDF provides reasonable approximations

$$P_{\text{LN}}(\delta)d\delta = \frac{1}{\sqrt{2\pi}\sigma} \exp \left[ - \frac{(\log \delta + \sigma^2/2)^2}{2\sigma^2} \right] \frac{d\delta}{\delta}$$

Lognormal Model from hydro simulations

Both the density and temperature fluctuations can be approximated by lognormal distribution:

\[
P(\delta_x; \sigma_{LN,x})d\delta_x = \frac{1}{\sqrt{2\pi \sigma_{LN,x}}} \exp \left( \frac{-(\log \delta_x + \sigma_{LN,x}^2/2)^2}{2\sigma_{LN,x}^2} \right) \frac{d\delta_x}{\delta_x}
\]
Each radial bin

Fluctuation (data)

Lognormal function

\[ \delta_T = \frac{T(r)}{\overline{T}(r)} \]

\[ \delta_n = \frac{n(r)}{\overline{n}(r)} \]
Application to a real cluster: A3667

Good agreement with the Lognormal distribution

Estimated value of

\[ \sigma_{\text{LN}, n} = 0.75 + \frac{50}{(\alpha_{\text{Sx}} - 0.2)^4} \]

\[ \sigma_{\text{LN}, \text{Sx}} \sim 0.4 \]

An analytic model for $T_{\text{spec}}/T_{\text{ew}}$

- Spherical polytropic $\beta$-model as global mean radial profiles
- Log-normal density and temperature fluctuations
  - Density and temperature correlations are ignored
  - Radius independent $\sigma$ assumed

$$\frac{T_{\text{sl}}}{T_{\text{ew}}} = \frac{T_{\text{sl}}^{\text{RP}}}{T_{\text{ew}}^{\text{RP}}} \exp(-1.25\sigma_{LN,T}^2)$$

- Explain simulation results well

The Hubble constant measurement using galaxy clusters

- **SZ**: primary distance indicator
- Assumption: the spherical isothermal $\beta$ model

$$d_A = L / \theta$$

$$H_0 = \frac{cz}{d_A(z)} \left[ 1 + \frac{2\lambda_0 - \Omega_0 - 6}{4} z + \mathcal{O}(z^2) \right]$$

$$\frac{-\Delta I}{I} \propto y \approx n T L$$

+ X-ray brightness
$$S_x \propto n^2 \Lambda_x(T) L$$

+ spectroscopic $T$
$$T = T_{\text{spec}}$$

length $L$
Isothermal $\beta$ -model fit by force

- Isothermal $\beta$ -model fit to polytropic density and temperature profiles

$$< n > (r) = n_0 \left[ \frac{1}{1 + (r / r_c)^2} \right]^{3\beta/2}$$

$$< T > (r) = T_0 \left[ < n > (r) / n_0 \right]^{\gamma^{-1}}$$

- Core radius estimated from X-ray + SZ

$$r_{c,iso\beta}(T_{spec}) = \frac{y(0)^2}{S_X(0)} \frac{m_e c^4 \Lambda(T_{spec})}{4\pi (\sigma_T k T_{spec})^2 (1 + z)^4} \frac{G(\beta_{fit})}{G(\beta_{fit} / 2)^2}$$

$$\beta_{fit} = \beta \frac{\gamma + 3}{4}$$
Analytic modeling of $H_0$ measurement

- Spherical polytropic $\beta$ -model as mean radial profiles
- Log-normal density and temperature fluctuations
- Still fit to the isothermal $\beta$ -model by force, and the estimated $H_0$ is biased as

\[
\int_{H,\text{polyLN}|\text{iso}\beta} f_H = \frac{H_{0,\text{est}}}{H_{0,\text{true}}} = \chi_\sigma \chi_T(T_{ew}) \frac{\chi_T(T_{\text{spec}})}{\chi_T(T_{ew})}
\]

**inhomogeneity**

\[
\chi_\sigma = \exp(\sigma_{LN,n}^2 - \sigma_{LN,T}^2 / 8) \approx (1.1 - 1.3)
\]

**non-isothermality**

\[
\chi_T(T_{ew}) = J(\beta, \gamma, r_c / r_{vir})^{1.5} \left[ \frac{G(\beta(\gamma + 3) / 8)}{G(\beta \gamma / 2)} \right]^2 \approx (0.8 - 1)
\]

**temperature bias**

\[
\frac{\chi_T(T_{\text{spec}})}{\chi_T(T_{ew})} \approx \left( \frac{T_{\text{spec}}}{T_{ew}} \right)^{1.5} \approx (0.8 - 0.9)
\]

Mean values are in good agreement with the analytic model.

Additional small bias expected due to non-sphericity of clusters even after averaging over l.o.s. angles.

Non-spherical effect: triaxial clusters

- Synthetic triaxial clusters (Jing & YS 2002) + polytropic $\beta$ + log-normal fluctuations

Skewed distribution due to the prolateness

Previous studies did not find the large bias because we set $T_{cl}=T_{ew}$ instead of $T_{spec}$ (Inagaki, Suginohara & YS 1995, Yoshikawa, Itoh & YS 1998), consistent with our results of the isothermal fit with $T_{ew}$.
Asymmetry in the estimated $H_0$

- Simulated Coma
- Isothermal ($T_{\text{cl}} = T_{\text{spec}}$)
- Isothermal ($T_{\text{cl}} = T_{\text{ew}}$)
- Polytropic

$f_H \equiv H_{0,\text{est}} / H_{0,\text{true}}$

- Simulation
- Oblate
- Prolate

- Distribution function of $H_{0,\text{est}}$ from many SZ clusters
- $\leftrightarrow$ Overall average shape of clusters (oblate or prolate)
Summary of theoretical predictions

- $H_{0,\text{est}}/H_{0,\text{true}} = 0.8-0.9$ from simulated clusters
- Analytic modeling of $H_0$ from the SZ effect
- $H_{0,\text{est}}/H_{0,\text{true}} = 0.8-0.9$ from simulated clusters is well explained by the combination of inhomogeneity and non-isothermality of ICM
- Is this consistent with the existing SZ observations?
$H_0$ estimated from the SZ effect

- **ROSAT+SZ:**
  - $60 \pm 3$ km/s/Mpc
    (Reese et al. 02)

- **Chandra+SZ**
  - $76.9^{+3.9}_{-3.4}^{+10.0}_{-8.0}$ km/s/Mpc
    (Bonamente et al. 06)

- **WMAP:**
  - $73 \pm 3$ km/s/Mpc
    (Spergel et al. 07)

Which is believable (if any at all !) ?
Same SZ but different X-ray data

- Reese et al. (2002)
  - 60 km/s/Mpc with ROSAT
- Bonamente et al. (2006)
  - 77 km/s/Mpc with Chandra
  - calibration data ver.3.1
- Chandra calibration data revision (2009)
  - Jan. 2009 ver.4.1: effective area of mirror
  - Dec. 2009 ver.4.2: ACIS (AXAF CCD Imaging Spectrometer) contamination model

Effective areas for different calibrations

Majority of clusters are observed with ACIS-I (front illuminated chips).

Ver.3.1 overestimates A, and thus T as well.
Spectroscopic temperatures

Relative to the latest calibration data (ver. 4.2)
- Ver. 3.1 overestimates T by 6%
- Ver. 4.1 underestimates T by 7%

X-ray emissivity

Angular diameter distances

\( \Omega_\Lambda = 0.73, \Omega_m = 0.27 \) assumed

The Hubble constant of each SZ cluster

$$\frac{H_{0,2}}{H_{0,1}} = \left( \frac{T_2}{T_1} \right)^2 \frac{\Lambda_2^{\text{eff}}(T_1)}{\Lambda_1^{\text{eff}}(T_1)} \frac{\Lambda_1^{\text{eff}}(T_1)}{\Lambda_2^{\text{eff}}(T_2)} \frac{A_1(E_{\text{fid}})}{A_2(E_{\text{fid}})}$$

($$\Omega_\Lambda = 0.73, \Omega_m = 0.27$$)

Abundances

Summary of comparison

Table 3. Compilation of Mean Ratios: Updated A2163 $N_H$

<table>
<thead>
<tr>
<th>parameter</th>
<th>3.1/4.2</th>
<th>4.1/4.2</th>
<th>3.1/B06</th>
<th>4.1/B06</th>
<th>4.2/B06</th>
<th>ASCA/4.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_e$</td>
<td>1.06 ± 0.05</td>
<td>0.93 ± 0.03</td>
<td>1.05 ± 0.11</td>
<td>0.92 ± 0.10</td>
<td>0.99 ± 0.11</td>
<td>0.98 ± 0.12</td>
</tr>
<tr>
<td>$Z$</td>
<td>1.08 ± 0.21</td>
<td>0.96 ± 0.04</td>
<td>1.16 ± 0.43</td>
<td>1.03 ± 0.31</td>
<td>1.08 ± 0.34</td>
<td>0.66 ± 0.28</td>
</tr>
<tr>
<td>$\Lambda_{\text{eff}}$</td>
<td>1.01 ± 0.01</td>
<td>1.01 ± 0.01</td>
<td>1.03 ± 0.07</td>
<td>1.03 ± 0.07</td>
<td>1.02 ± 0.07</td>
<td>...</td>
</tr>
<tr>
<td>$f_{(\nu,T_e)}$</td>
<td>0.998 ± 0.002</td>
<td>1.002 ± 0.001</td>
<td>0.999 ± 0.003</td>
<td>1.003 ± 0.004</td>
<td>1.001 ± 0.003</td>
<td>...</td>
</tr>
<tr>
<td>$A(1\text{keV})^a$</td>
<td>1.01 ± 0.02</td>
<td>0.95 ± 0.01</td>
<td>0.96 ± 0.10</td>
<td>0.91 ± 0.10</td>
<td>0.95 ± 0.10</td>
<td>...</td>
</tr>
<tr>
<td>$d_A^b$</td>
<td>0.93 ± 0.08</td>
<td>1.13 ± 0.06</td>
<td>1.06 ± 0.24</td>
<td>1.29 ± 0.29</td>
<td>1.15 ± 0.25</td>
<td>1.07 ± 0.37</td>
</tr>
</tbody>
</table>

*a* B06 are the effective areas from the 3.1 calibration using only those data sets that appear in Bonamente et al. 2006.

*b* B06 are the published distances from Bonamente et al. 2006.

- Systematic difference between different calibration data of Chandra

## Compilation of \(H_0\) results

<table>
<thead>
<tr>
<th>(H_0)</th>
<th>DL A2163 (N_H)</th>
<th>Updated A2163 (N_H)</th>
<th>No A2163</th>
</tr>
</thead>
<tbody>
<tr>
<td>(H_0^{3.1})</td>
<td>82.8 ± 4.6 (75.9)</td>
<td>70.0 ± 3.7 (40.6)</td>
<td>69.7 ± 3.7 (40.5)</td>
</tr>
<tr>
<td>(H_0^{4.1})</td>
<td>58.4 ± 3.1 (42.4)</td>
<td>55.4 ± 2.9 (34.3)</td>
<td>55.5 ± 2.9 (34.3)</td>
</tr>
<tr>
<td>(H_0^{4.2})</td>
<td>68.8 ± 3.7 (52.2)</td>
<td>63.7 ± 3.3 (38.8)</td>
<td>63.7 ± 3.4 (38.8)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(\chi^2) R02 overlap</th>
<th>17</th>
<th>17</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>(H_0^{3.1})</td>
<td>90.1 ± 7.0 (48.5)</td>
<td>66.9 ± 4.7 (15.8)</td>
<td>66.1 ± 4.8 (15.5)</td>
</tr>
<tr>
<td>(H_0^{4.1})</td>
<td>58.2 ± 4.1 (21.2)</td>
<td>52.3 ± 3.6 (11.8)</td>
<td>52.2 ± 3.8 (11.8)</td>
</tr>
<tr>
<td>(H_0^{4.2})</td>
<td>70.1 ± 5.1 (28.7)</td>
<td>60.5 ± 4.3 (14.4)</td>
<td>60.2 ± 4.4 (14.3)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Avg Full sample</th>
<th>38</th>
<th>38</th>
<th>37</th>
</tr>
</thead>
<tbody>
<tr>
<td>(H_0^{3.1})</td>
<td>66.1 ± 30.3</td>
<td>62.9 ± 21.7</td>
<td>62.6 ± 21.9</td>
</tr>
<tr>
<td>(H_0^{4.1})</td>
<td>52.5 ± 17.2</td>
<td>51.3 ± 15.4</td>
<td>51.2 ± 15.6</td>
</tr>
<tr>
<td>(H_0^{4.2})</td>
<td>59.7 ± 22.1</td>
<td>58.0 ± 18.9</td>
<td>57.8 ± 19.1</td>
</tr>
</tbody>
</table>

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<thead>
<tr>
<th>Avg R02 overlap</th>
<th>17</th>
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<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>(H_0^{3.1})</td>
<td>68.3 ± 36.1</td>
<td>61.2 ± 17.6</td>
<td>60.4 ± 17.7</td>
</tr>
<tr>
<td>(H_0^{4.1})</td>
<td>51.8 ± 16.9</td>
<td>49.2 ± 12.0</td>
<td>48.9 ± 12.3</td>
</tr>
<tr>
<td>(H_0^{4.2})</td>
<td>60.0 ± 23.3</td>
<td>56.1 ± 15.5</td>
<td>55.6 ± 15.9</td>
</tr>
</tbody>
</table>

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<thead>
<tr>
<th>(\chi^2) B06 refit</th>
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<th>37</th>
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<tr>
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<td>...</td>
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<th>(\chi^2) R02 refit</th>
<th>18</th>
<th>17</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>(H_0^{R02})</td>
<td>60.8 ± 4.0 (16.5)</td>
<td>60.5 ± 4.1 (16.4)</td>
<td>60.7 ± 4.3 (16.4)</td>
</tr>
</tbody>
</table>

- **DL:** Dickey & Lockman (1990)
- **R02:** Reese et al. (2002)
- **B06:** Bonamente et al. (2006)

\[
\Omega_\Lambda = 0.73
\]
\[
\Omega_m = 0.27
\]
assumed
## Ups and downs of $H_0$ from SZ+Xray

<table>
<thead>
<tr>
<th>X-ray data</th>
<th>$H_0$ [km/s/Mpc]</th>
<th>reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROSAT+ASCA</td>
<td>$60 \pm 3$</td>
<td>Reese et al. (2002)</td>
</tr>
<tr>
<td>Chandra: ver. 3.1</td>
<td>$77^{+3.9}_{-3.4}$</td>
<td>Bonamente et al. (2006)</td>
</tr>
<tr>
<td>WMAP</td>
<td>$73 \pm 3$</td>
<td>Spergel et al. (2007)</td>
</tr>
<tr>
<td>Chandra: ver. 3.1</td>
<td>$70.0 \pm 3.7$</td>
<td>this work</td>
</tr>
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<td>Chandra: ver. 4.1</td>
<td>$55.4 \pm 2.9$</td>
<td>this work</td>
</tr>
<tr>
<td>Chandra: ver. 4.2</td>
<td>$63.7 \pm 3.3$</td>
<td>this work</td>
</tr>
</tbody>
</table>

Conclusions

- X-ray calibration is not robust as believed before
  - Cluster temperature may vary ±7%
  - $H_0$ combined with SZ may vary ±12%
- If the latest Chandra calibration data (ver.4.2) is the most reliable, $H_0(SZ) \sim 0.9H_0(WMAP)$
  - This might indicate the presence of the inhomogeneities in intra-cluster medium (Kawahara et al. 2008a)
- Possible systematics for cluster cosmology in general
  - Mass-temperature relation of clusters
  - Cluster abundances and $\sigma_8$
- The goal of cluster cosmology is not to find a good (but boring) agreement with WMAP+others. Look for inconsistency!